Scalar perturbations in f(R)-cosmology in the late universe

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- **f(R)-cosmologies**
- **astrophysical and cosmological approach**
- **quasistatic approximation**

$$S = \frac{1}{2\kappa^2}\int \sqrt{-g}\, f(R)\, d^4x + S_m,$$

where S_m is action for matter and $\kappa^2 = 8\pi G$.

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T_{\mu\nu},$$

$$3\Box F(R) + F(R)R - 2f(R) = \kappa^2 T$$

$$F(R)R - 2f(R) = 0,$$

de Sitter points

$$f(R) = f(R_{dS}) + F(R_{dS})(R - R_{dS}) + o(R - R_{dS}) =$$
$$= -f(R_{dS}) + 2\frac{f(R_{dS})}{R_{dS}}R + o(R - R_{dS})$$

These models go asymptotically to the de Sitter space when $R \to R_{dS} \neq 0$ with
$\Lambda = \frac{R_{dS}}{4}$

$$f(R) = R - 2\Lambda + o(R - R_{dS})$$

Several examples of functions $f(R)$, which have de Sitter points:

$$f(R) = R^2$$

$$f(R) = R - \mu^2 \frac{c_1 (\frac{R}{\mu^2})^k + c_3}{c_2 (\frac{R}{\mu^2})^k + 1}$$

$$f(R) = R - a[\tanh(\frac{b(R - R_0)}{2}) + \tanh(\frac{bR_0}{2})]$$

In the case of the spatially flat background spacetime with the metrics

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) ,$$

the Hubble parameter $H = \dot{a}/a$ **and the scalar curvature**

$$R = 6(2H^2 + \dot{H})$$

These equations give :

$$3FH^2 = (FR - f)/2 - 3H\dot{F} + \kappa^2\bar{\rho},$$

$$-2F\dot{H} = \ddot{F} - H\dot{F} + \kappa^2(\bar{\rho} + \bar{P})$$

where the perfect fluid with the energy-momentum tensor components $T^{\mu}_{\nu} = \mathrm{diag}(-\rho, P, P, P)$ **satisfies the continuity equation**

$$\dot{\bar{\rho}} + 3H(\bar{\rho} + \bar{P}) = 0$$

Now let us turn to the formula (6.1) from [Antonio De Felice], describing the perturbed metric, and without loss of generality present it in the following form:

$$ds^2 = -(1+2\alpha)dt^2 + a^2(1+2\psi)\delta_{ij}dx^i dx^j$$

CONFORMAL NEWTONIAN GAUGE

$$-\frac{\Delta\Psi}{a^2}+3H\left(H\Phi+\dot{\Psi}\right)=-\frac{1}{2F}[\left(3H^2+3\dot{H}+\frac{\Delta}{a^2}\right)\delta F-$$

$$-3H\delta\dot{F}+3H\dot{F}\Phi+3\dot{F}\left(H\Phi+\dot{\Psi}\right)+\kappa^2\delta\rho]\,,$$

$$H\Phi+\dot{\Psi}=\frac{1}{2F}\left(\delta\dot{F}-H\delta F-\dot{F}\Phi\right)\,,-F(\Phi-\Psi)=\delta F\,,$$

$$3\left(\dot{H}\Phi+H\dot{\Phi}+\ddot{\Psi}\right)+6H\left(H\Phi+\dot{\Psi}\right)+3\dot{H}\Phi+\frac{\Delta\Phi}{a^2}=$$

$$=\frac{1}{2F}[3\delta\ddot{F}+3H\delta\dot{F}-6H^2\delta F-\frac{\Delta\delta F}{a^2}-3\dot{F}\dot{\Phi}-3\dot{F}\left(H\Phi+\dot{\Psi}\right)-$$

$$-\left(3H\dot{F}+6\ddot{F}\right)\Phi+\kappa^2(\delta\rho+\delta P)]\,,$$

$$\delta\ddot{F} + 3H\delta\dot{F} - \frac{\Delta\delta F}{a^2} - \frac{1}{3}R\delta F = \frac{1}{3}\kappa^2(\delta\rho - 3\delta P) +$$

$$+\dot{F}(3H\Phi + 3\dot{\Psi} + \dot{\Phi}) + 2\ddot{F}\Phi + 3H\dot{F}\Phi - \frac{1}{3}F\delta R\,,$$

$$\delta F = F'\delta R,$$

$$\delta R = -2[3\left(\dot{H}\Phi + H\dot{\Phi} + \ddot{\Psi}\right) + 12H\left(H\Phi + \dot{\Psi}\right) +$$

$$+\frac{\Delta\Phi}{a^2} + 3\dot{H}\Phi - 2\frac{\Delta\Psi}{a^2}]$$

So, the previous system of equations describes the scalar cosmological perturbations in the case of the nonlinear $f(R)$ theory of gravity.

Once again: we will consider the Universe at the late stage of its evolution (when galaxies are formed) and deep inside the cell of uniformity 150 **Mpc**
We will investigate the astrophysical approach in the case of Minkowski spacetime background and two cases in the cosmological approach:

- **large scalaron mass approximation**
- **quasi-static approximation**

We will get explicit expressions for scalar perturbations for both these cases.

In the mechanical approach, galaxies can be considered as separate compact objects with rest mass density

$$\rho = \frac{1}{a^3} \sum_i m_i \delta(\vec{r} - \vec{r}_i) \equiv \frac{\rho_c}{a^3}$$

$$\delta\rho = \rho - \overline{\rho} = \frac{\rho_c - \overline{\rho_c}}{a^3}$$

Smallness of non-relativistic gravitational potentials Φ and Ψ , and smallness of peculiar velocities are two independent conditions! We will work in two steps:

- **we neglect peculiar velocities and we define gravitational potential**
- **then we use this potential to determine dynamical behaviour of galaxies**

This gives us the possibility to take into account both the gravitational attraction between inhomogeneities and the global cosmological expansion of the Universe. This presentation is about the first step in the program.

Astrophysical approach: we neglect all time derivatives and we have Minkowski spacetime background

$$-\frac{\Delta}{a^2}\Psi = -\frac{1}{2F}\left(\frac{\Delta}{a^2}\delta F + \kappa^2\delta\rho\right) ,$$

$$-F(\Phi - \Psi) = \delta F ,$$

$$\frac{\Delta}{a^2}\Phi = \frac{1}{2F}\left(-\frac{\Delta}{a^2}\delta F + \kappa^2\delta\rho\right) ,$$

$$-\frac{\Delta}{a^2}\delta F = \frac{1}{3}\kappa^2\delta\rho - \frac{1}{3}F\delta R ,$$

$$\delta F = F'\delta R, \quad \delta R = -2\left(\frac{\Delta}{a^2}\Phi - 2\frac{\Delta}{a^2}\Psi\right)$$

$$\Psi = \frac{1}{2F}\delta F + \frac{\varphi}{a} = \frac{F'}{2F}\delta R + \frac{\varphi}{a}$$

$$\Phi = -\frac{1}{2F}\delta F + \frac{\varphi}{a} = -\frac{F'}{2F}\delta R + \frac{\varphi}{a}$$

$$\Delta\varphi = \frac{1}{2F}\kappa^2 a^3 \delta\rho = \frac{1}{2F}\kappa^2 \delta\rho_c = \frac{4\pi G_N \delta\rho_c}{F}, \quad G_N = \frac{\kappa^2}{8\pi F}$$

HELMHOLTZ EQUATION

$$\Delta\delta R - \frac{a^2}{3}\frac{F}{F'}\delta R = -\frac{a^2}{3F'}\frac{\kappa^2}{F}\delta\rho$$

$$M^2 = \frac{a^2 F}{3F'}$$

Now we will do a cosmological approach. This means that the background functions may depend on time. It is hardly possible to solve the system directly. Therefore we study first the case of the very large mass of the scalaron:

$$-\frac{\Delta\Psi}{a^2} + 3H\left(H\Phi + \dot{\Psi}\right) = -\frac{1}{2F}\left[3H\dot{F}\Phi + 3\dot{F}\left(H\Phi + \dot{\Psi}\right)\right],$$

$$H\Phi + \dot{\Psi} = \frac{1}{2F}\left(-\dot{F}\Phi\right),$$

$$\Phi - \Psi = 0\,,$$

$$3\left(\dot{H}\Phi + H\dot{\Phi} + \ddot{\Psi}\right) + 6H\left(H\Phi + \dot{\Psi}\right) + 3\dot{H}\Phi + \frac{\Delta\Phi}{a^2} =$$

$$= \frac{1}{2F}\left[-3\dot{F}\dot{\Phi} - 3\dot{F}\left(H\Phi + \dot{\Psi}\right) - \left(3H\dot{F} + 6\ddot{F}\right)\Phi + \kappa^2\delta\rho\right],$$

$$0 = \dot{F}(3H\Phi + 3\dot{\Psi} + \dot{\Phi}) + 2\ddot{F}\Phi + 3H\dot{F}\Phi\,,$$

$$0 = \left(\dot{H}\Phi + H\dot{\Phi} + \ddot{\Psi}\right) + 4H\left(H\Phi + \dot{\Psi}\right) + \frac{\Delta\Phi}{3a^2} + \dot{H}\Phi - \frac{2}{3}\frac{\Delta\Psi}{a^2}$$

$$\Psi = \Phi = \frac{\varphi}{a\sqrt{F}}$$

$$\frac{\Delta\varphi}{a^3\sqrt{F}} + \frac{3\dot{F}^2\varphi}{4aF^2\sqrt{F}} = \frac{\kappa^2\delta\rho}{2F}$$

$$F(R) = 1 + o(1),$$

$$f(R) = R - 2\Lambda + o(R - R_\infty)$$

Thus, in the case of large enough scalaron mass we reproduce the ”linear” cosmology from the ”non-linear” one, as it should be.

Quasistatic approximation:

$$M^2 = \frac{a^2}{3}\left(\frac{F}{F'} - \frac{R}{R_{dS}}\right)$$

$$\Psi = \frac{F'}{2F}\left[\frac{\kappa^2}{12\pi F'}\sum_i \frac{m_i \exp(-M|\vec{r}-\vec{r}_i|)}{|\vec{r}-\vec{r}_i|} - \frac{\kappa^2}{(F - F' R_{dS})a^3}\overline{\rho_c}\right] + \frac{\varphi}{a}$$

$$\Phi = \frac{-F'}{2F}\left[\frac{\kappa^2}{12\pi F'}\sum_i \frac{m_i \exp(-M|\vec{r}-\vec{r}_i|)}{|\vec{r}-\vec{r}_i|} - \frac{\kappa^2}{(F - F' R_{dS})a^3}\overline{\rho_c}\right] + \frac{\varphi}{a}$$

$$3H\left(H\Phi+\dot{\Psi}\right)=-\frac{1}{2F}[\left(3H^2+3\dot{H}+\frac{\Delta}{a^2}\right)\delta F-3H\delta\dot{F}+$$
$$+3H\dot{F}\Phi+3\dot{F}\left(H\Phi+\dot{\Psi}\right)],$$

$$H\Phi+\dot{\Psi}=\frac{1}{2F}\left(\delta\dot{F}-H\delta F-\dot{F}\Phi\right),$$

$$3\left(\dot{H}\Phi+H\dot{\Phi}+\ddot{\Psi}\right)+6H\left(H\Phi+\dot{\Psi}\right)+3\dot{H}\Phi+\frac{\Delta\Phi}{a^2}=$$
$$=\frac{1}{2F}[3\delta\ddot{F}+3H\delta\dot{F}-6H^2\delta F-\frac{\Delta\delta F}{a^2}-$$
$$3\dot{F}\dot{\Phi}-3\dot{F}\left(H\Phi+\dot{\Psi}\right)-\left(3H\dot{F}+6\ddot{F}\right)\Phi],$$

$$\delta\ddot{F} + 3H\delta\dot{F} - \frac{\Delta\delta F}{a^2} = \dot{F}(3H\Phi + 3\dot{\Psi} + \dot{\Phi}) + 2\ddot{F}\Phi + 3H\dot{F}\Phi \,,$$

$$\frac{F'}{F}R_{dS}\delta R = -2[3\left(\dot{H}\Phi + H\dot{\Phi} + \ddot{\Psi}\right) + \\ + 12H\left(H\Phi + \dot{\Psi}\right) + \frac{\Delta\Phi}{a^2} + 3\dot{H}\Phi - 2\frac{\Delta\Psi}{a^2}]$$

$$\delta F = F'\delta R$$

Conclusions: In our work , we have studied scalar perturbations in non-linear $f(R)$-gravity.
The main objective was to find explicit expressions for Φ and Ψ in the framework of nonlinear $f(R)$ models. In the case of nonlinearity, the system of equations for scalar perturbations is very complicated. It is hardly possible to solve it directly. Therefore we have considered the following approximations: the astrophysical approach, the large scalaron mass case in cosmological approximation and the quasistatic approximation also in the cosmological approach. In all three cases, we found the explicit expressions for the scalar perturbation functions Φ and Ψ up to required accuracy.

The quasi-static approximation is of most interest from the point of view of the large scale structure investigations. Here, the gravitational potential Φ contains both the nonlinearity function F and the scale factor a. Hence we can study the dynamical behaviour of the inhomogeneities including into consideration their gravitational attraction and the cosmological exapansion, and also taking into account the effect of nonlinearity. All this make it possible to carry out the numerical and analytical analysis of the large scale structure dynamics in the late Universe for $f(R)$ models.

This presentation was prepared according to works:

- arXiv 1401.5401, J.N., M. Eingorn, A.Zhuk
- Hubble flows and gravitational potentials in observable Universe, M. Eingorn, A.Zhuk
- $f(R)$ - theories, A. De Felice, S. Tsujikawa