Testing gravity theories using tensor perturbations

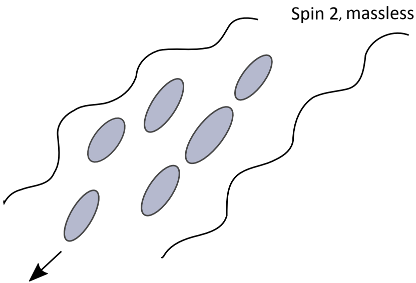
Jan Novák

Technical university in Liberec, Czech republic

7.9.2017, Praha

Graviton

Central role in quantization of the gravitational field is played by the graviton, which is a massless particle of spin 2.

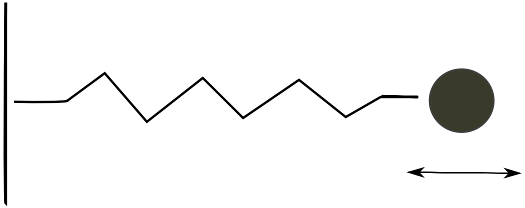


Propagation equation

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \tag{1}$$

$$\ddot{h}_k + 3\frac{\dot{a}}{a}\dot{h}_k + \frac{a^2}{k^2}h_k = 16\pi G\Pi_k^T, \tag{2}$$

Harmonic oscillator



Relativistic theories of gravity other than GR can

- change the damping rate of the graviational waves in the second term,
- or they can modify the speed of propagation of the gravitational waves in the third term
- or add some source term on the RHS.[1]

[1] At second order in perturbations anisotropic stress always appears even when the matter consists completely of dust. It means that there will be a RHS term at second order in perturbations.

Modified propagation equation

$$h_k'' + 2\frac{g'}{g}h_k' + k^2 h_k = F_k(\eta), \tag{3}$$

where we included all terms on the RHS in one function $F_k(\eta)$.

Gravitational perturbation inside horizon

$$h_k'' + 2\frac{g'}{g}h_k' + k^2 h_k = 0 \tag{4}$$

When we use the substitution $W = gh_k$, we obtain this equation

$$W'' + (k^2 - \frac{a''}{a})W = 0. \tag{5}$$

We obtain the solution

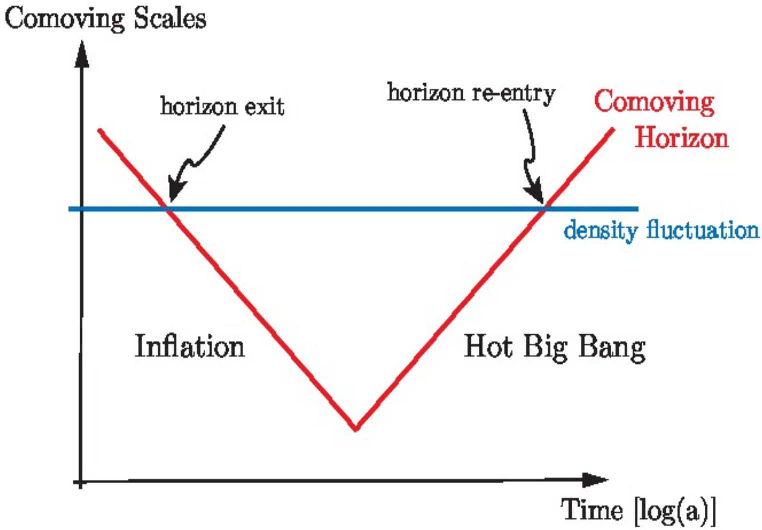
$$h_k(t) \to \frac{\sqrt{16\pi G}}{(2\pi)^{3/2}\sqrt{2kg}} \exp(-ik\int d\eta) \tag{6}$$

in early times of inflation.

Horizon

Testing gravity theories using tensor perturbations

Jan Novák



Gravitational perturbation outside horizon

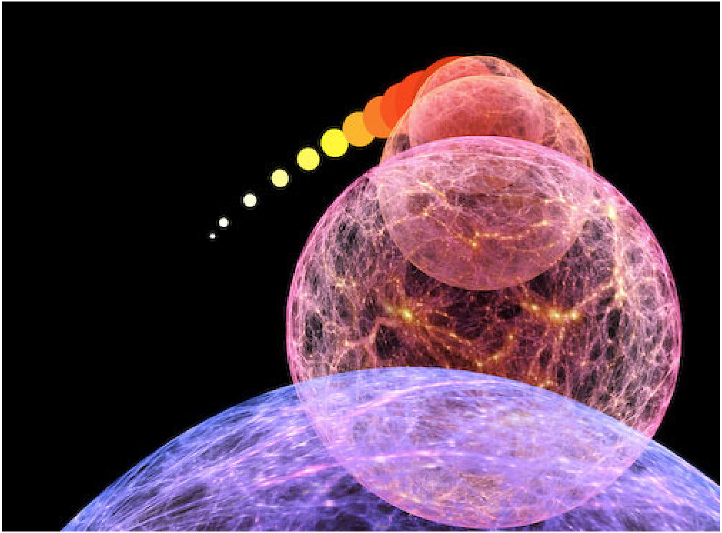
Under this assumption, we have $a = -\frac{1}{H\eta}$, where H is constant expansion rate during inflation. And the above equation (4) becomes

$$h_k'' - 2\frac{(1+\tilde{\nu}_0)}{\tau}h_k' + k^2 h_k = 0. \tag{7}$$

If we let $x = -k\eta$ and $h_k = x^{\frac{3}{2}+\tilde{\nu}_0}y$, the above equation becomes

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + [x^2 - (\frac{3}{2} + \tilde{\nu}_0)^2]y = 0. \tag{8}$$

Inflation



Tensor mode spectrum

Bessel equation

$$t^2\ddot{y} + t\dot{y} + [t^2 - \nu^2]y = 0 \tag{9}$$

From this we obtain the tensor mode spectrum

$$\Delta_t = \frac{G(2H)^{2(1+\tilde{\nu}_0)}[\Gamma(\frac{3}{2} + \tilde{\nu}_0)]^2}{\pi^3 k^{3+2\tilde{\nu}_0}}. \tag{10}$$

Inflation consistency equation

Since we get the dependence $k^{-3-2\tilde{\nu}_0}$ we can identify the tensor spectral index as

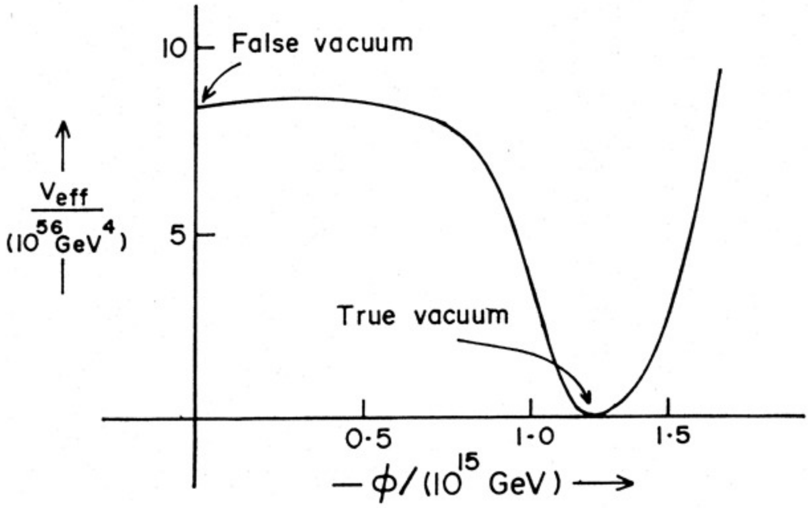
$$n_T = -2\tilde{\nu}_0. \qquad (11)$$

It vanishes in GR. Let's look at slow roll inflation. We obtain the result

$$n_T = -2\tilde{\nu}_0 - 2\epsilon, \qquad (12)$$

where $\epsilon = -\frac{\dot{H}}{H^2}$.

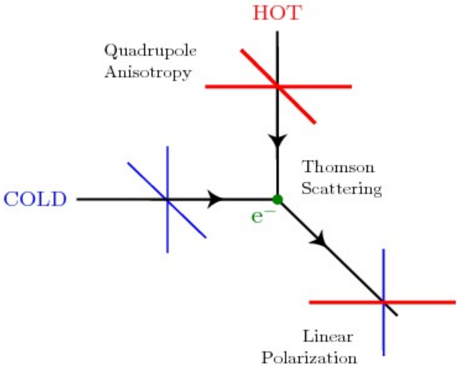
Inflation


10
False vacuum
5
Veff
(10^56 GeV^4)
True vacuum
0·5
1·0
1·5
— φ/(10^15 GeV) →

Modified inflation consistency equation

$$n_T = -2\tilde{\nu}_0 - r/8, \tag{13}$$

$\tilde{\nu}_0$ affects the CMB B-mode power spectrum



Anisotropic stress is a sign of a modification of gravity!

Source term on the RHS:

$$a^2\Gamma\gamma_{ij}, \; \Gamma = \Gamma(\frac{a}{b}, a_i) \tag{14}$$

Pictures were taken from www.scienceblogs.com, Inflation in brane world gravity (A.Banersee) and TASI lectures on Inflation (D.Baumann).