Scalar perturbations of Galileon cosmologies in the mechanical approach in the late Universe

Perturbation theory as a probe of viable cosmological models

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Logo of Technical University in Liberec

Founded 1953.

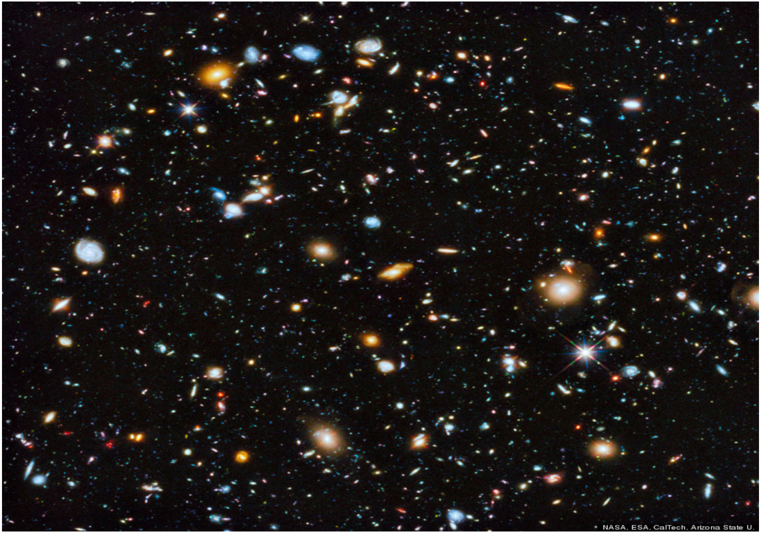
Outline

- Mechanical approach
- Coupled scalar fields: the mechanical approach and the late cosmic acceleration
- Galileon fields in the mechanical approach in the late Universe

Mechanical approach

- hydrodynamical approach is in strongly nonlinear regime inapplicable
- then N-body simmulation is commonly used
- it is important to define theoretically at which scales we should perform transition from the highly inhomogeneous mechanical distribution to the smooth hydrodynamical one

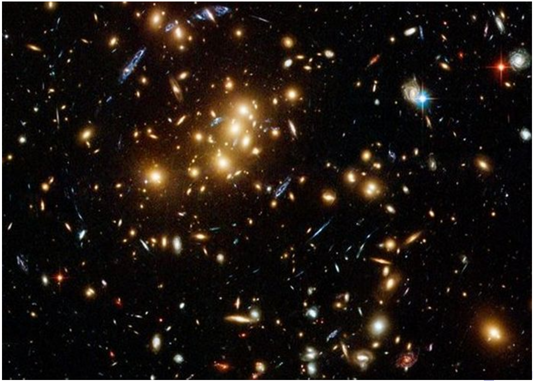
When we consider the cell of uniformity (150-370 Mpc). There are discrete distributed inhomogeneities.



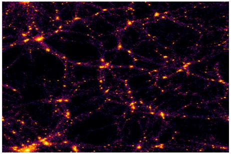
- discrete structures, which perturb the background Friedmann model
- gravitational potentials for an arbitrary number of randomly distributed inhomogeneities within the ΛCDM
- equations from the first principles, we can generalize our analysis on various alternative cosmological models and we can check their compatibility with observations
- goal of the mechanical approach is to select viable models (ΛCDM, $f(R)$-theories, Chaplygin gas, quintessence models, Chevalier-Polarski-Linder)

Scalar perturbations of FLRW Universe

We consider the stage of the Universe evolution, which is much latter than the recombination time. At this stage, the formation of inhomogeneities has been generally completed. The cell of statistical homogeneity/uniformity size is of the order of 150 Mpc.



On much bigger scales the Universe is well described by the ΛCDM model with matter mainly in the form of dark matter



plus the cosmological constant.

Here, dark matter is well simulated by a pressurless perfect fluid and the hydrodynamical approach provides the adequate description of the model.

ΛCDM background model and mechanical approach

We start with the energy-momentum tensor of non-interacting randomly distributed particles (inhomogenities, in our case):

$$T^{ik} = \sum_p \frac{m_p c^2}{(-g)^{1/2}[\eta]} \frac{dx^i}{ds} \frac{dx^k}{ds} \frac{ds}{d\eta} \delta(\vec{r} - \vec{r}_p) =$$

$$\sum_p \frac{m_p c^2}{(-g)^{1/2}[\eta]} \frac{dx^i}{d\eta} \frac{dx^k}{d\eta} \frac{d\eta}{ds} \delta(\vec{r} - \vec{r}_p),$$

where m_p is the mass of the p-th inhomogeneity and $[t]$ and $[\eta]$ indicate that the determinant is calculated from the metric coefficients defined with respect to synchronous t or conformal η times.

In the ΛCDM model, the main contributions come from the cosmological constant and the nonrelativistic matter. Therefore, the peculiar velocities should be much less than the speed of light :

$$\frac{dx^{\alpha}}{d\eta} = a\frac{dx^{\alpha}}{dt}\frac{1}{c} << 1$$

Therefore, we can assume that T^{00} is the only non-zero component of the energy-momentum tensor:

$$T^{00} = \sum_p \frac{m_p c^2}{\sqrt{-g_{00} g}} \delta(\vec{r} - \vec{r}_p) = \frac{\sqrt{\gamma} \rho c^2}{\sqrt{-g_{00} g}},$$

where γ is the determinant of the metrics $\gamma_{\alpha\beta}$ and we introduce the rest mass density

$$\rho = \frac{1}{\sqrt{\gamma}} \sum_p m_p \delta(\vec{r} - \vec{r}_p).$$

After averaging T^{00} over all space, we get $\bar{T}^{00} = \bar{\rho}c^2/a^5$, where $\bar{\rho}$ is the average rest mass density ρ and we use the unperturbed metrics. Therefore $\bar{T}^0_{\ 0} = \bar{\rho}c^2/a^3$.

The inhomogeneities in the Universe result in scalar perturbations of the metrics. In the conformal Newtonian gauge, such perturbed metrics is

$$ds^2 \approx a^2[(1+2\Phi)d\eta^2 - (1-2\Psi)\gamma_{\alpha\beta}dx^\alpha dx^\beta],$$

where scalar perturbations depend on all space-time coordinates η, x, y, z and satisfy equations ...

$$\Delta\Psi - 3H(\Psi' + H\Phi) + 3K\Psi = \frac{1}{2}\kappa a^2 \delta T^0_0$$

$$\frac{\partial}{\partial x^\beta}(\Psi' + H\Phi) = \frac{1}{2}\kappa a^2 \delta T^0_\beta = 0$$

$$[\Psi'' + H(2\Psi + \Phi)' + (2H' + H^2)\Phi + \frac{1}{2}\Delta(\Phi - \Psi) - K\Psi]\delta^\alpha_\beta$$
$$-\frac{1}{2}\gamma^{\alpha\sigma}(\Phi - \Psi)_{;\sigma;\beta} = -\frac{1}{2}\kappa a^2 \delta T^\alpha_\beta = 0$$

The condition $\delta T^0_\beta = 0$ follows from the **nonrelativistic nature of the considered matter**, $|\delta T^0_\beta| << \delta T^0_0$, and we can drop δT^0_β with respect to δT^0_0. To clarify this point, we want to stress that according to previous equations, both δT^0_0 and δT^0_β contribute to the gravitational potential Φ. However, due to the previous condition, peculiar velocities of inhomogenities are nonrelativistic and the contribution of δT^0_β is negligible compared to that of δT^0_0. In other words, account of δT^0_β is beyond accuracy of the model.

Following the standard argumentation, we can put $\Phi = \Psi$, then the system of above equations reads

$$\Delta\Phi - 3H(\Phi' + H\Phi) + 3K\Phi = \frac{1}{2}\kappa a^2 \delta T^0_0,$$
$$\frac{\partial}{\partial x^\beta}(\Phi' + H\Phi) = 0,$$
$$\Phi'' + 3H\Phi' + (2H' + H^2)\Phi - K\Phi = 0.$$

So, from the second equation we get

$$\Phi(\eta, \vec{r}) = \frac{\varphi(\vec{r})}{c^2 a(\eta)},$$

where $\varphi(\vec{r})$ is a function of all spatial coordinates and we have introduced c^2 for convenience.

We shall see that $\varphi(\vec{r}) \sim \frac{1}{r}$ in the vicinity of an inhomogeneity, and the non-relativistic gravitational potential $\Phi(\eta, \vec{r}) \sim \frac{1}{ar} = \frac{1}{R}$, where $R = ar$ is the physical distance. Φ has the correct Newtonian limit near inhomogeneities. We get

$$\Delta\varphi + 3K\varphi = \frac{1}{2}\kappa c^2 a^3 \delta T^0_0.$$

Our final master equation is

$$\Delta\varphi + 3K\varphi = 4\pi G_N(\rho - \bar{\rho}).$$

ρ and $\bar{\rho}$ are **comoving local** and **average rest mass densities**, which do not depend on time.

Annual conference in Odessa

Odessa theater

Scalar fields

We want to gain a model for the accelerated expansion of the Universe!

ΛCDM, $w = -1$

Very popular are now models using scalar fields:

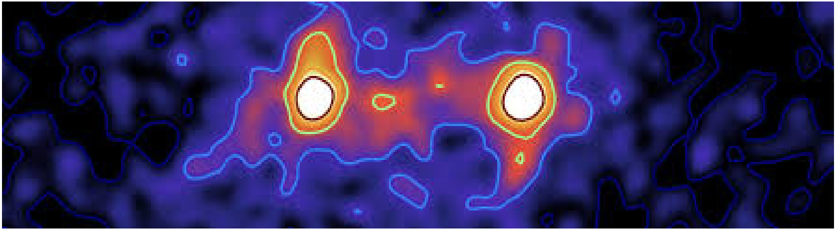
- $-1 < w < 0$, quintessence
- $w < -1$, phantom
- $w = -1$ crossing, quintom

Theory of cosmological perturbations is a powerful tool to **investigate cosmological models.**

- the mechanical approach works perfectly for the ΛCDM model where the **peculiar velocities** of the **inhomogeneities** can be considered as **negligibly small**
- we consider the scales deep inside the cell of uniformity
- we drop the peculiar velocities at the first order approximation
- such an approach was generalized also to the case of cosmological models with different perfect fluids, which can play a role of dark energy and matter

- **fluctuations** of the **additional perfect fluids** also form their own **inhomogeneities**
- in the mechanical approach it is supposed that the velocities of displacement of such inhomogeneities is of the order of the peculiar velocities of inhomogeneities of dust like matter, they are nonrelativistic; in some sense these type of inhomogeneities are coupled to each other

They can play an important role of dark matter and can be distributed around the baryonic inhomogeneities in such way that it can solve the problem of **flatness of rotation curves.**



- cosmological model with a scalar field minimally coupled to gravity
- the universe is also filled with dust-like matter (in the form of discrete galaxies and group of galaxies) and radiation
- we study the theory of scalar perturbations for such a model and obtain a condition under which the inhomogeneities of dust like matter and the inhomogeneities of the scalar field can be coupled to each other
- first, the coupled scalar field behaves as a 2-component perfect fluid: a cosmological constant and a network of frustrated cosmic strings

- the potential of such scalar field is very flat at the present time
- this flatness condition is natural consequence of the current acceleration of the universe, as the contribution of the term with $w = -\frac{1}{3}$ has to be very small at present
- the fluctuations of the scalar field are absent and the energy density and pressure of the scalar field fluctutate due to the interaction of the gravitational potential with the scalar field background

- such a coupled scalar field is in concordance with the theory of scalar perturbations and contributes to the gravitational potential
- the fluctuations of the energy density of the scalar field are concentrated around the galaxies, screening their gravitational potential
- such a distribution of the energy density of the scalar field fluctuations justifies the coupling conditions

FLRW: homogeneous and isotropic

$$ds^2 = a^2(\eta)(d\eta^2 - \gamma_{\alpha\beta}dx^\alpha dx^\beta)$$

Scalar field minimally coupled to gravity:

$$S_\phi = \int \sqrt{-g}(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi))$$

Background energy density and pressure:

$$\bar{T}^0_0 \equiv \bar{\epsilon}_\varphi = \frac{1}{2a^2}(\phi_c')^2 + V(\phi_c)$$
$$-\bar{T}^i{}_i \equiv \bar{p}_\varphi = \frac{1}{2a^2}(\phi_c')^2 - V(\phi_c)$$

As matter sources, we also include dust-like matter and radiation.

$$H^2 = \frac{\kappa a^2}{3}[\bar{\epsilon}_{DUST} + \bar{\epsilon}_{RAD} + \frac{1}{2}\frac{(\phi_c')^2}{a^2} + V(\phi_c)] - K,$$

$$H' = \frac{1}{3}a^2\kappa[-\bar{\epsilon}_{RAD} - \frac{1}{2}\bar{\epsilon}_{DUST} - \frac{(\phi_c')^2}{a^2} + V(\phi_c)],$$

$$\kappa = \frac{8\pi G_N}{c^4}.$$

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)\gamma_{\alpha\beta}dx^\alpha dx^\beta],$$

$$\delta T^0_0 \equiv \delta\epsilon_\varphi = -\frac{1}{a^2}(\phi_c')^2\Phi + \frac{1}{a^2}\phi_c'\varphi' + \frac{dV}{d\phi}(\varphi_c)\varphi,$$

$$\delta T^0_i = \frac{1}{a^2}\phi_c'\partial_i\varphi,$$

$$\delta T^i_j \equiv -\delta^i_j\delta p_\varphi,$$

$$\delta p_\varphi = -\frac{1}{a^2}(\phi_c')^2\Phi + \frac{1}{a^2}\phi_c'\varphi' - \frac{dV}{d\phi}(\phi_c)\varphi,$$

$$\phi = \phi_c + \varphi.$$

$$
\begin{gathered}
\Delta\Phi - 3H(\Phi' + H\Phi) + 3K\Phi = \frac{\kappa}{2}a^2(\delta\epsilon_{dust} + \delta\epsilon_{rad}) \\
-\kappa/2[(\phi_c')^2\Phi - \phi_c'\varphi' - a^2\frac{dV}{d\phi}(\phi_c)\varphi], \\
\partial_i\Phi' + H\partial_i\Phi = \frac{\kappa}{2}\phi_c'\partial_i\varphi, \\
\frac{2}{a^2}[\Phi'' + 3H\Phi' + \Phi(2\frac{a''}{a} - H^2 - K)] = \\
= \kappa[\delta p_{rad} - \frac{1}{a^2}(\phi_c')^2\Phi + \frac{1}{a^2}\phi_c'\varphi' - \frac{dV}{d\phi}(\phi_c)\varphi].
\end{gathered}
$$

- according to the **mechanical approach**, we **drop** the terms containing the **peculiar velocities of the inhomogeneities and radiation** as these are **negligible** when compared with their respective energy density and pressure fluctuations
- such comparison with respect to the scalar field is not evident since the quantity treated as the peculiar velocity of the scalar field is proportional to the scalar field perturbation

- first we preserve the scalar field perturbation; then the subsequent analysis of the equations must show whether or not we can equate to zero the RHS of the equation

$$\partial_i \Phi' + H \partial_i \Phi = \frac{\kappa}{2} \phi_c' \partial_i \varphi$$

- we shall demonstrate that for the coupled scalar field the RHS of this equation can indeed be set to zero in a consistent way within the mechanical approach as it usually happens for coupled fluids

$$\Phi' + H\Phi = \frac{\kappa}{2}\phi_c'\varphi$$

$$\Phi[H' - H^2 - K + \kappa\frac{1}{2}(\phi_c')^2] = \varphi[-\frac{\kappa}{2}\phi_c'' - H\kappa\phi_c' - \frac{a^2}{2}\kappa\frac{dV}{d\phi}(\phi_c)] + \\ + \kappa\frac{a^2}{2}\delta p_{rad},$$

where we have also used the relation $\frac{2a''}{a} = 2(H' + H^2)$.

$$\Phi[-\frac{2}{3}a^2\kappa\bar{\epsilon}_{rad} - \frac{1}{2}a^2\kappa\bar{\epsilon}_{dust}] = \kappa\frac{a^2}{2}\delta p_{rad}$$

$$\delta p_{rad} = -\Phi\bar{\epsilon}_{dust} = -\Phi\frac{\bar{\rho}c^2}{a^3} = \frac{1}{3}\delta\epsilon_{rad}$$

$$\delta\epsilon_{dust} = \frac{\delta\rho c^2}{a^3} + 3\frac{\bar{\rho}\Phi}{a^3}$$

$$\delta\rho = \rho - \bar{\rho}$$

$$\Delta\Phi - 3H(\Phi' + H\Phi) + 3K\Phi =$$
$$\frac{\kappa}{2}\frac{\delta\rho c^2}{a} - \frac{\kappa}{2}[(\phi_c')^2\Phi - \phi_c'\varphi' - a^2\frac{dV}{d\phi}(\phi_c)\varphi]$$

We get

$$\varphi = \frac{\Phi' + H\Phi}{\frac{\kappa}{2}\phi_c'},$$

$$\varphi' = \frac{\Phi'' + H'\Phi + H\Phi'}{\frac{\kappa}{2}\phi_c'} - \frac{\Phi' + H\Phi}{\frac{\kappa}{2}(\phi_c')^2}\phi_c''.$$

$$\Delta\Phi = -\frac{\kappa}{2}\frac{\Delta\rho c^2}{a} = \Phi[3H^2 - 3K - \frac{\kappa}{2}(\phi_c')^2 + H' - H\frac{\phi_c''}{\phi_c'} +$$
$$+a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] + \Phi'[4H - \frac{\phi_c''}{\phi_c'} + a^2\frac{dV}{d\phi}(\phi_c)\frac{1}{\phi_c'}] + \Phi''.$$

Since

$$\Phi' = \frac{d\Phi}{da}aH,\ \Phi'' = \frac{d^2\Phi}{da^2}a^2H^2 + \frac{d\Phi}{da}aH' + \frac{d\Phi}{da}aH^2.$$

After the substitution of $\Phi = \Omega/a$, where Ω is a function of a and the spatial coordinates, we get

$$\frac{\Delta\Omega}{a} - \frac{\kappa}{2}\frac{\delta\rho c^2}{a} = -\frac{\Omega}{a}[3K + \frac{\kappa}{2}(\phi_c')^2] +$$
$$+\frac{d\Omega}{da}[3H^2 + H' - H\frac{\phi_c''}{\phi_c'} + a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] + \frac{d^2\Omega}{da^2}aH^2.$$

Dust like matter is considered in the form of discrete distributed inhomogeneities. Then we are looking for solutions which have Newtonian limit near gravitating masses

$$\Omega = \Omega(\vec{r})$$

in agreenment with the transition to the astrophysical approach

$$a \to const. \Rightarrow H \to 0$$

and all background energy densities are equal to zero, and we should select the flat topology $K = 0$.

If the dust like matter is described by the discrete gravitational sources with masses m_i and the rest mass density

$$\rho = \sum_i m_i \delta(\vec{r} - \vec{r}_i),$$

the gravitational potential Φ is

$$\Phi = -\frac{G_N}{c^2}\frac{1}{a}\sum_i \frac{m_i}{|\vec{r} - \vec{r}_i|} = -\frac{G_N}{c^2}\sum_i \frac{m_i}{|\vec{R} - \vec{R}_i|},$$

as it should be.

In the previous equation we took into account the realations between the physical and comoving radius vectors: $\vec{R} = a\vec{r}$; This equation also demonstrates that $\Phi \sim \frac{1}{a}$.

Now, we analyse the case $\Omega = \Omega(\vec{r}) \Rightarrow \Phi \sim \frac{1}{a}$ and $(\phi_c')^2 = const.$ in more details. Let us denote $\phi_c' = \beta = const.$. Then we get

$$\phi_c = \beta\eta + \gamma, \gamma = const.$$

The substitution of this equation into eqution of motion gives:

$$2\frac{a'}{a}\beta + a^2\frac{dV}{d\phi}\phi_c = 2\frac{a'}{a}\beta + a^2\frac{V'}{\beta} = 0$$
$$\Rightarrow V = \frac{\beta^2}{a^2} + V_\infty,\ V_\infty = const.$$

$$\bar{\epsilon}_\varphi = \frac{3}{2}\frac{\beta^2}{a^2} + V_\infty$$
$$\bar{p}_\varphi = -\frac{1}{2}\frac{\beta^2}{a^2} - V_\infty$$

$$\Phi \sim \frac{1}{a},\ \Phi' + H\Phi = 0 \Rightarrow \varphi = \varphi' = 0$$

The physical reason for this is that the "coupling" between the inhomogeneities of the dust like matter and of the scalar field imposes strong restriction on the scalar field!
The above analysis demonstrates that such a scalar field can exist.

On the other hand, the fluctuations of the energy density and pressure of the scalar field are non-zero:

$$\delta\epsilon_\varphi = \delta p_\varphi = -\frac{1}{a^2}(\phi_c')^2\Phi = -\frac{\beta^2}{a^3}\Omega(\vec{r}) \neq 0$$

These fluctuations arise due to the interaction between the scalar field background and the gravitational potential. Previous equation shows that

$$\delta\epsilon_\varphi \sim \frac{1}{a^3}$$

in analogy also with the fluctuations of the energy density for a perfect fluid with the constant equation of state parameter $w = -\frac{1}{3}$.

- We can prove that the fluctuations of the energy density of the scalar field contribute to the gravitational potential.
- The fluctuations of the density of the scalar field are concentrated around the inhomogeneities of the dust like matter (around the galaxies) which is in the full agreenment with the coupling condition. The presence of these fluctuations leads to the screening of the gravitational potential.

K-essence fields

$$S_p = \int \sqrt{|g|} L_p = \int \sqrt{|g|} P(X, \phi),$$

$$X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,$$

$$\frac{1}{\sqrt{|g|}} \partial_\mu (\sqrt{|g|} g^{\mu\nu} P_X \partial_\nu \phi) - P_\phi = 0$$

Galileon

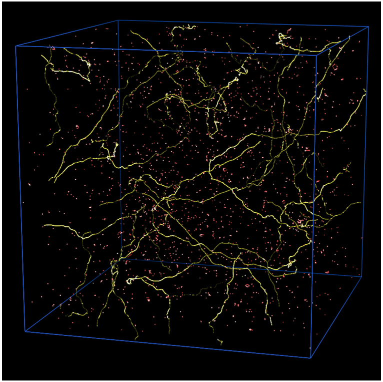
Mostly inspired by **DGP models**, people derived the five Lagrangians that lead to field equations invariant under the Galileon symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in the Minkowski spacetime:

$$
\begin{gathered}
L_1 = M^3\phi,\ L_2 = (\nabla\phi)^2,\ L_3 = (\Box\phi)(\nabla\phi)^2/M^3, \\
L_4 = (\nabla\phi)^2[2(\nabla\phi)^2 - 2\phi_{;\mu\nu}\phi^{;\mu\nu} - R(\nabla\phi)^2/2]/M^6, \\
L_5 = (\nabla\phi)^2[(\nabla\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}^{\ \nu}\phi_{;\nu}^{\ \rho}\phi_{;\rho}^{\ \mu} \\
-6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}]/M^9,
\end{gathered}
$$

The scalar field that repsects the **Galileon symmetry** is the Galileon.

The result for coupled Galileon field

At the background level, such Galileon field behaves as a 3-component perfect fluid: a network of **cosmic strings** with the EoS parameter $w = -\frac{1}{3}$, **cosmological constant** and some **matter component.**



Action

$$S_I = \alpha \int_M \sqrt{|g|} \Box\phi \, \partial_\mu \phi \partial^\mu \phi \, d^4x,$$

α is a small parameter, which measure the deviation from the model of **minimally coupled scalar field** and his units L^3 (L is a length). First we will compute the tensor of energy momentum for this Lagrangian by the following formula:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

$g^{\mu\nu}$ will be the following matrix:

$$\begin{bmatrix} \frac{1-2\Phi}{a^2} & 0 & 0 & 0 \\ 0 & -\gamma^{11}\frac{1+2\Psi}{a^2} & -\gamma^{12}\frac{1+2\Psi}{a^2} & -\gamma^{13}\frac{1+2\Psi}{a^2} \\ 0 & -\gamma^{21}\frac{1+2\Psi}{a^2} & -\gamma^{22}\frac{1+2\Psi}{a^2} & -\gamma^{23}\frac{1+2\Psi}{a^2} \\ 0 & -\gamma^{31}\frac{1+2\Psi}{a^2} & -\gamma^{32}\frac{1+2\Psi}{a^2} & -\gamma^{33}\frac{1+2\Psi}{a^2} \end{bmatrix}$$

We use the case $K = 0$ and so $\gamma^{ij} = \gamma_{ij}$ is equal to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the trace $\gamma = \gamma^{ij}\gamma_{ij}$.

D'Alembertian

We compute $\Box\phi$ with the perturbed quantities. We use the notation $\phi = \phi_c + \varphi$.

$$\Box\phi = \frac{\phi_c''}{a^2} - \frac{2}{a^2}\phi_c''\Phi + \frac{\varphi''}{a^2} - \frac{\Delta_{ij}\varphi}{a^2}$$
$$-\frac{\gamma}{a^2}\phi_c'(\frac{a'}{a} - \Psi' - 2\Phi\frac{a'}{a}) - \frac{a'}{a}\frac{\gamma}{a^2}\varphi' - \frac{a'\phi_c'}{a^3} - \frac{\varphi' a'}{a^3} +$$
$$+\frac{4\Phi\phi_c' a'}{a^3} - \frac{\phi_c'}{a^3}(\Phi' a + 2a'\Phi) - \Box_{ij}\varphi\frac{1}{a^2}$$

Complete action

Now we wrote the tensor of energy-momentum for whole action, when we include also the minimally coupled scalar field:

$$S = \int_M \sqrt{|g|}(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V(\phi) + \alpha\Box\phi\partial_\mu\phi\partial^\mu\phi)\, d^4x,$$

where α is a small parameter, which measure the deviation from the model of minimally coupled scalar field and it has units L^3 (L is length).

Einstein equations

So, the first Einstein equation is the following

$$\Delta\Phi - 3H(\Phi' + H\Phi) + 3K\Phi = \frac{\kappa}{2}a^2(\delta\epsilon_{dust} + \delta\epsilon_{rad}) +$$

$$+\frac{\kappa}{2}[-(\phi_c')^2\Phi + \phi_c'\varphi' + a^2\frac{dV}{d\phi}(\phi_c)\varphi +$$

$$2\alpha\gamma\frac{(\phi_c')^2}{a^2}(\phi_c'\Psi' + 4\Phi\frac{a'}{a}\phi_c' - 3\varphi'\frac{a'}{a}) -$$

$$-\alpha 2\frac{(\phi_c')^2}{a^2}\Box_{ij}\varphi]$$

EoM

We use now the equation of motion

$$-2\alpha(\Box\phi)^2 + 2\alpha\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi + 2\alpha\nabla^\mu\phi\nabla^\nu\phi R_{\mu\nu} -$$

$$\Box\phi - \frac{dV}{d\phi} = 0$$

Peculiar velocity of the scalar field

When we consider the mechanical approach, we can **drop** the terms containing the **peculiar velocities** of the inhomogeneities and radiation as these are **negligible** when compared with their respective **energy density** and **pressure fluctuations.** If we deal with scalar field, such an approach is not evident since the quantity treated as the peculiar velocity of the scalar field is proportional to the scalar field perturbation φ.

Work with Einstein equations

$$\frac{\delta\epsilon_{rad}}{3} = \frac{-1}{3a^6}[(36\alpha H\phi_c'\phi_c'' + 18\alpha H^2(\phi_c')^2 a^2 + 18\alpha H'(\phi_c')^2 a^2 +$$
$$+12\alpha\phi_c'' a\phi_c' a' + 18\alpha(\phi_c')^2 a'' a -$$
$$-18\alpha(\phi_c')^2(a')^2)\varphi - 30\alpha\Phi(\phi_c')^3 a' a - 6\alpha\Phi(\phi_c')^2\phi_c'' a^2 +$$
$$(36\alpha(\phi_c')^2 a' a)\varphi' + 3\Phi a^6\bar{\epsilon}_{dust} + 4\Phi a^6\bar{\epsilon}_{rad}]$$

As matter sources, we also include dust-like matter (baryonic and CDM) and radiation. The background (average) energy density of the dust-like matter takes the form $\bar{\epsilon}_{dust} = \bar{\rho}c^2/a^3$, where $\bar{\rho} = const$ is the average comoving rest mass density. For radiation we have the EoS $\bar{p}_{rad} = \frac{1}{3}\bar{\epsilon}_{rad}$ and $\epsilon_{rad} \sim 1/a^4$.

Pure scalar field

$$\Phi[-\frac{2}{3}a^2\kappa\bar{\epsilon}_{rad} - \frac{1}{2}a^2\kappa\bar{\epsilon}_{dust}] = \kappa\frac{a^2}{2}\delta p_{rad}$$

$$\delta p_{rad} = -\Phi\bar{\epsilon}_{dust} = -\Phi\frac{\bar{\rho}c^2}{a^3} = \frac{1}{3}\delta\epsilon_{rad}$$

Central equation for pure scalar field

Next we make the substitution $\Phi = \Omega/a$ in the following equation for pure scalar field:

$$\Delta\Phi = -\frac{\kappa}{2}\frac{\delta\rho c^2}{a} = \Phi[3H^2 - 3K - \frac{\kappa}{2}(\phi_c')^2 + H' - H\frac{\phi_c''}{\phi_c'} +$$

$$+a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] + \frac{d\Phi}{da}a[5H^2 + H' - H\frac{\phi_c''}{\phi_c'} + a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] +$$

$$\frac{d^2\Phi}{da^2}H^2a^2.$$

Application of mechanical approach

The **dust like matter** component is considered in the form of **discrete distributed inhomogeneities.** Then we are looking for solutions of previous equation, which have **Newtonian limit** near gravitating masses. Such an asymptotic behaviour will take place if we impose $\Omega = \Omega(\vec{r})$.

Cosmological constant and cosmic strings

$\Omega = \Omega(\vec{r}) \Rightarrow \Phi \sim \frac{1}{a}$ and $(\phi_c')^2 = const.$
Let's denote $\phi_c' = \beta = const.$, then we get

$$\phi_c = \beta\eta + \gamma, \gamma = const,$$

$$V = \frac{\beta^2}{a^2} + V_\infty.$$

Galileon cosmologies and mechanical approach

Now. we suppose that $\Omega = \Omega(r)$, which means that $\phi_c' = const.$:

$$\phi_c(\eta) = \beta\eta + \omega,$$

where ω and β are constants. Then we get from EoM that

$$6\alpha\beta^2 a'' + 2a^2 a'\beta + \frac{dV}{d\phi} a^5 = 0$$

Potential for Galileon cosmologies

We want to obtain the dependence $f(\eta)$ in the relation

$$V(\eta) = \frac{\beta^2}{a^2} + V_\infty + \alpha f(a),$$

because we know the dependence $V(a) = \frac{\beta^2}{a^2} + V_\infty$ for the pure scalar field.
$f(a)$ behaves like matter:

$$f(a) \sim \frac{1}{a^3}$$

Articles

This presentation was prepared with the help of

- Maxim Eingorn, J.N., Alexander Zhuk, f(R) gravity: scalar perturbations in the late Universe, EPJ C,2014, arXiv:1401.5410
- Mariam Bouhmadi-Lopéz, J.N., Coupled scalar perturbations of Galileon cosmologies in the mechanical approach in the late Universe, in preparation
- Alexander Zhuk, Perfect fluids coupled to inhomogeneities in the late Universe, arXiv: 1601.01939
- Alvina Burgazli, Alexander Zhuk, Joao Morais, Mariam Bouhmadi Lopéz, K.Sravan Kumar, Coupled scalar fields in the late Universe: The mechanical appraoch and the late time cosmic acceleration, arXiv: 1512.03819v3

Thank You for listening!

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Sources of pictures: Arizona State University, Backreaction: blogger, Discovery magazine blog, NASA Getty Images, The University of Chicago, Cosmology: Brian Koberlein, New Scientist