Causal set approach to quantum gravity and one mathematical problem

Jan Novák

Department of physics
Technical University in Liberec

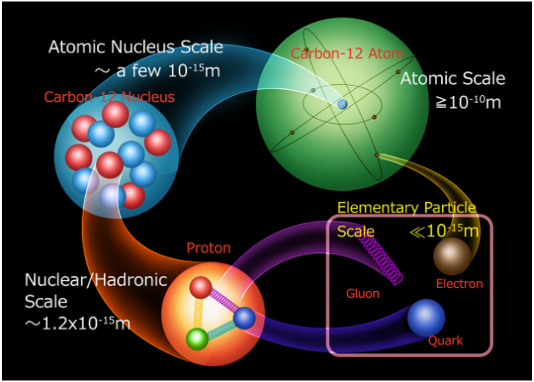
Bilbao, EPV/EHU, 20.6.2018

CHARLES
UNIVERSITY
IN PRAGUE

Logo of Technical University in Liberec

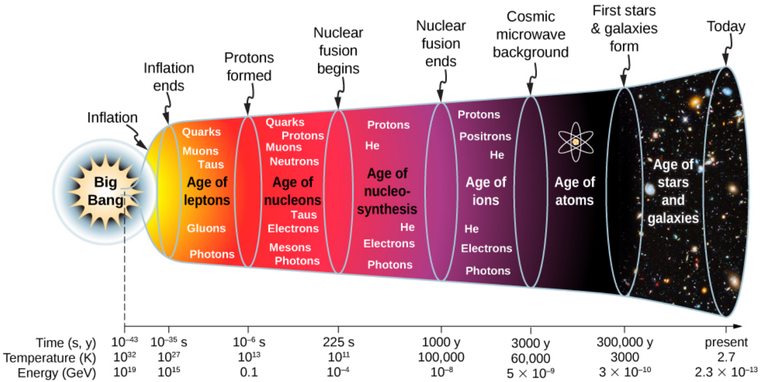
Founded 1953.

We are made of ...



- molecules
- atoms
- elementary particles
- something else??

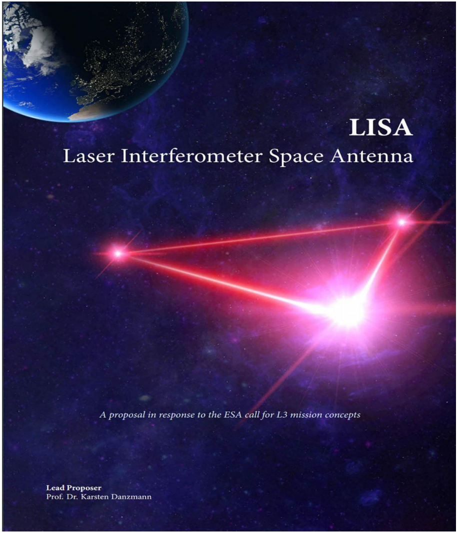
Hot Big Bang Scenario



Accelerated expansion of the Universe

The Universe is expanding in the last 5 billion years.

LISA, TAIJI, TIANQIN



First second of the evolution

- Quark-gluon plasma, 10^{-5} s
- Baryogenesis, 10^{-10} s
- Grand Unification, 10^{-36} s
- Inflation, $10^{-43} - 10^{-32}$ s

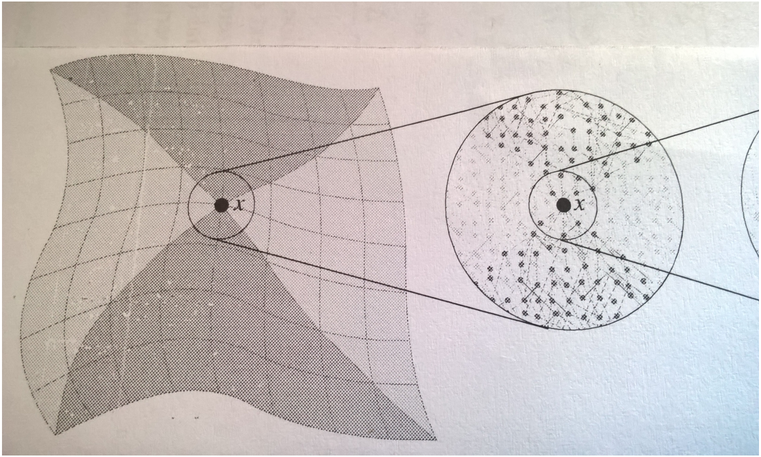
Dark energy after GW170817

$$|\frac{c_g}{c} - 1| \leq 5.10^{-16}$$

Large class of scalar-tensor theories and dark energy models are highly disfavored.

Why quantum gravity?

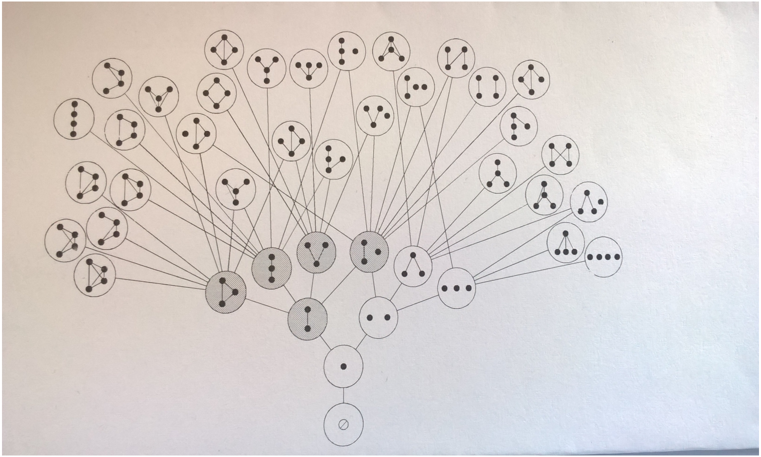
- That GR cannot be true at the most fundamental level is clear from singularity theorems.
- The universal coupling of gravity to all forms of energy would make it plausible that gravity has to be implemented in a quantum framework too.
- Quantum theory and GR contain a drastically different concept of time. They are incompatible.


x
x

Quantum Universe

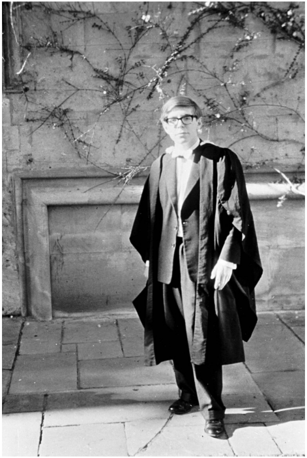
1. Higher level structure (quantum)
2. Lower level structure (classical)

- Not distinguish between on-shell and off-shell.
- Generalized Hasse diagrams.



Causal metric hypothesis

The properties of the physical Universe are manifestations of causal structure.



Causal metric hypothesis

The causal structure of relativistic spacetime determines its metric structure up to scale:

- Rafael Sorkin: order plus number equals geometry
- Stephen Hawking: topological structure determines conformal structure
- David Malament: causal structure determines topological structure

topological, smooth, causal, conformal, metric

Axioms of irreflexive formulation

The irreflexive formulation of causal set theory is defined by the following six axioms:

1. Binary axiom
2. Measure axiom
3. Countability
4. Transitivity
5. Interval finitness
6. Irreflexivity

Basic ideas

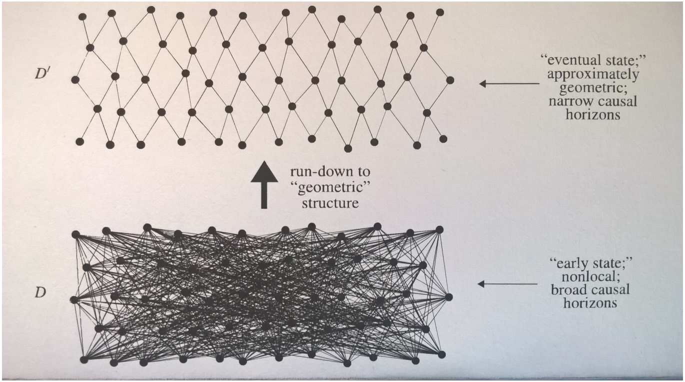
- Experimental bounds on Lorentz invariance violation do not present a serious obstacle to the development of discrete causal theory.
- Symmetry is much less central than generally believed notion of covariance.
- Problem of recovering a classical history from its relation space looks very much like boundary value problem.

Notion of spacetime

- Spacetime is a part what happens, not merely a place where things happen.
- GR is not perfectly background independent.
- Spacetime, particles and fields as aspects of something more fundamental.
- Emergent aspects of spacetime: particle

Cosmological inflation

- Inflation: what really happened in the early Universe is that ST ran down from relatively random causal structure, to sparser but more regular structure
- Why would causal structure grow sparser?
- Why would it stabilize in "geometric" structure?



D′
"eventual state;"
approximately
geometric;
narrow causal
horizons
run-down to
"geometric"
structure
D
"early state;"
nonlocal;
broad causal
horizons

Causal set approach

- Describing fundamental spacetime structure.
- Modeling gravitation at the quantum level.
- Unifying physical laws.

One of the results: heuristic bound on the value of cosmological constant in concordance with the experiment

The number of elements N comprising a region of spacetime equals the volume of that region in fundamental units. ($geometry = order + number$) The equality between number N and volume V is not precise, but a subject to Poisson fluctuations, whence instead of $N = V$, we can write only

$$N \sim V \pm \sqrt{V}.$$

Let us assume that at sufficiently large scales the effective theory of spacetime structure is governed by a gravitational path integral, which at a deeper level will of course be a sum over causets. That N plays the role of time in this sum suggest that it must be held fixed. If we were to fix V exactly, we would be doing "umimodular gravity", in which setting it is easy to see that Λ **and** V **are conjugate** to each other in the same sense as energy and time are conjugate in nonrelativistic quantum mechanics.

In analogy to the $\Delta E \Delta t$ uncertainty relation, we thus expect in quantum gravity to obtain

$$\Delta \Lambda \Delta V \sim \hbar.$$

Remember that even with N held exactly constant, V still fluctuates between $N + \sqrt{N}$ and $N - \sqrt{N}$. That is, we have $N \sim V \pm \sqrt{N} \rightarrow V \sim N \pm \sqrt{V}$, or $\Delta V \sim \sqrt{V}$.

$$\Delta \Lambda \sim \sqrt{V}^{-\frac{1}{2}}$$

Finally let us assume that the value about which Λ fluctuates is strictly zero: $\langle \Lambda \rangle = 0$. A rough estimate identifying spacetime volume with the Hubble scale H^{-1} then yields

$$V \sim (H^{-1})^4 \sim H^{-4} \rightarrow \Lambda \sim V^{-\frac{1}{2}} \sim H^2 \sim \rho_{critical}.$$

Related theories to causal set approach

1. Causal dynamical triangulation
2. Category theoretic approaches
3. Quantum automatons
4. Tensor networks
5. Causal nets
6. Domain theory
7. Quantum information theory
8. Loop quantum gravity
9. Twistor theory
10. Shape dynamics

Continuum based theories

1. Divergence issues
2. Lack of natural scale
3. Experimental discreteness
4. Discreteness arising from continuum based assumptions
5. Discreteness via the philosophy of measurement

Chain

Let $M = (M, R, i, t)$ be a multidirected set.

1. A chain γ in M is a sequence of elements and relations of the form $... \prec x_0 \prec x_1 \prec ...$ in M, where the notation $x_n \prec x_{n+1}$ refers to particular relation r in R such that $x_n = i(r)$ and $x_{n+1} = t(r)$. The chain set $Ch(M)$ of M is the set of all chains in M.
2. A chain of length n, or n-chain, between x and y in M, is a chain γ of the form $x = x_0 \prec x_1 \prec ... \prec x_{n-1} \prec x_n = y$. The element x is called the initial element of γ and the element y is called the terminal element of γ. The set of n-chains $Ch_n(M)$ in M is the subset of $Ch(M)$ consisting of all chains of length n. A complex chain is a chain of length at least two.

Antichain

1. A cycle in M is a chain $x_0 \prec x_1 \prec \ldots \prec x_{n-1} \prec x_n$ of nonzero length such that $x_0 = x_n$; its initial element coincides with its terminal element.
2. A relation r in R is called reducible, if there exists a complex chain from its initial element to its terminal element. Such a chain is called a reducing chain for r. If r is not reducible, it is called irreducible. M itself is called irreducible if all its relations are irreducible
3. An antichain σ in M is a subset of M admitting no chain of nonzero length in M between any pair of elements x and y in σ, distinct or otherwise.

Lemma

Lemma

Let $M = (M, R, i, t)$ *be a multidirected set, and let* σ *be an antichain in* M*. Suppose that* x *is an element of* M *belonging neither to* σ *itself, nor to the past or future of* σ*, nor to a cycle in* M*. Then the subset* $\sigma' = \sigma \cup \{x\}$ *of* M *is an antichain in* M*.*

Definition

Let $M = (M, R, i, t)$ be a multidirected set, interpreted as a model of information flow or causal structure, and let x and y be elements of M.

1. A family Γ of chains between x and y in M is called dependent if there exists another such family Γ', not containing Γ encoding all information or causal influence encoded by Γ.
2. In particular, a chain γ from x to y in M is called dependent if there exists a family Γ' of chains from x to y, not including γ encoding all information or causal influence encoded by γ.
3. If a chain or family of chains is not dependent, it is called independent.

Six arguments against transitivity

1. Multiple independent modes of influence between pairs of events are ubiquitous in conventional physics.
2. Independence of influences exerted by an event should not be constrained by details of its future.
3. Irreducibility and independence of relations between pairs of elements are a priori distinct conditions.
4. Configuration spaces of transitive binary relations are pathological, particularly from a physical perspective.
5. Structural notions from mathematics motivate the existence of independent modes of influence.
6. Recognition of nontransitive relations leads naturally to other improvements in discrete causal theory.

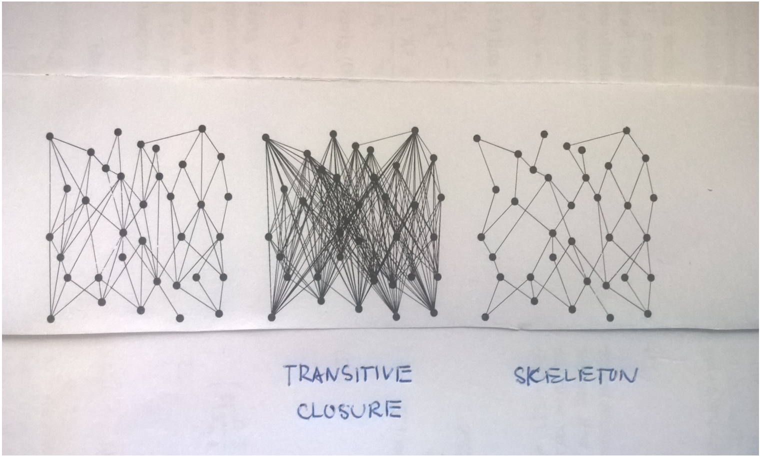
Definition

Let $D = (D, \prec)$ be a directed set, viewed as a model of causal structure under the independence convention. In this context, the binary relation $\prec$ on D is called the causal relation on D.

Definition

Let $D = (D, \prec)$ be a directed set.

1. The transitive closure of D is the directed set $tr(D) \equiv (D, \prec_{tr})$ whose binary relation $\prec_{tr}$ is defined by setting $x \prec_{tr} y$ if and only if there exists a chain of nonzero length between x and y in D. The binary relation $\prec_{tr}$ is called the transitive relation on D.
2. The skeleton of D is the acyclic directed set $sk(D) \equiv (D, \prec_{tr})$ whose binary relation $\prec_{sk}$ is defined by setting $x \prec_{sk} y$ if and only if $x \prec y$ is an irreducible relation in D. The binary relation $\prec_{sk}$ is called the skeletal relation on D.


TRANSITIVE
CLOSURE
SKELETON

Star finite and interval finite causal set

Interval finitness does not imply star finitness! An example is an infinite bouquet.
Star finitness does not imply interval finitness. An example is the Jacob's ladder.
A typical causal set defined via global sprinkling into $\mathbb{R}^{3+1}$, is star infinite at every element .

New version of axioms

A new version of discrete classical causal theory may be defined by:

1. Binary axiom
2. Generalized measure axiom
3. Countability
4. Star finitness
5. Acyclicity

Emergent particles

- emerge from discrete causal structure without the necessity of importing auxiliary mathematical content as Hilbert spaces
- Spatially localized family of events that retains similar internal structure over time interval.
- Emergent aspect of ST, rather than as "separate entities" existing on ST, can lead to possible insights into famous problems as the magnitude of the cosmological constant and the nature of dark matter.

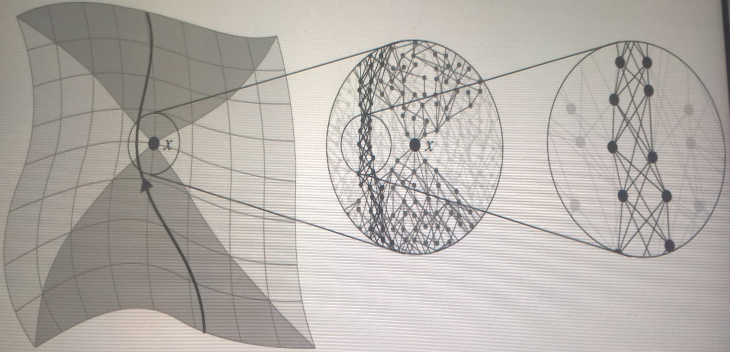
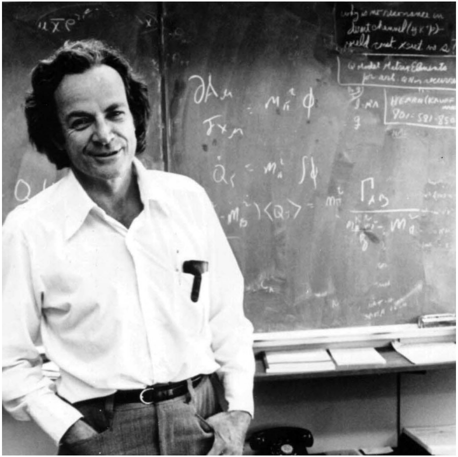


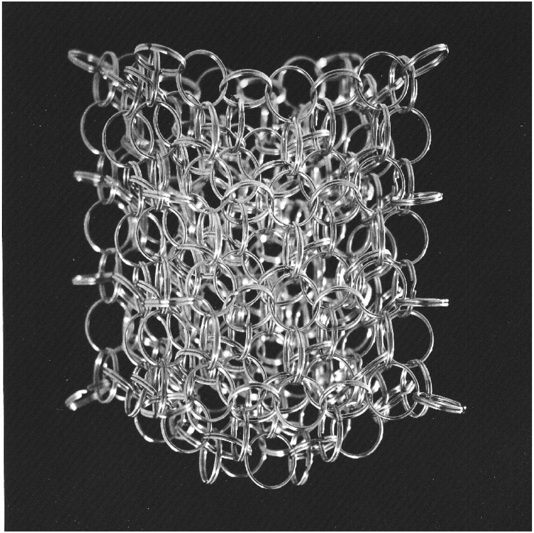
Fig. 1.7.3 A "particle trajectory" in a classical history.

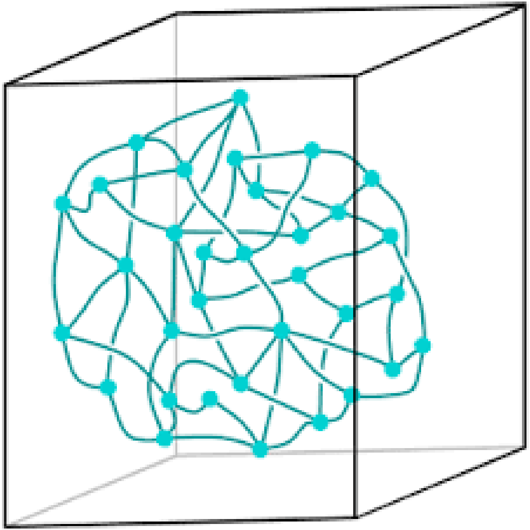
Poincare group

- the symmetry properties of Minkowski ST $\mathbb{R}^{3+1}$ are crucial to every area of physics that incorporates special relativity
- we need to find the emergence of P : how "near-symmetries" including Poincare symmetries may arise from causal structure at the fundamental scale

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are or what your name is. If it doesn't agree with an experiment, it is wrong.







One mathematical problem from topology

We formulate one mathematical problem, which will be useful further. We will call, that a circle $S^1 \in \mathbb{R}^3$ with finite length and finite circumference, which could be deformed, is a ring:
let's have a finite collection of N rings S^1 in $\mathbb{R}^3$, which could not touch. Derive a formula for number of non-homeomorphic structures, which could be constructed from this finite collection of rings; Every two rings could be linked only once, they could not be knotted or twisted.

Partial solution of the problem

We have immediately one bound from below on the **number of non-homeomorphic structures**, when we map the linkage of rings to finite connected graphs on N vertices. We simply exchange two rings, which are Hopf-linked by two vertices connected by an edge. So the number of linkage of N rings is at least so big as the number of connected graphs on N vertices. This is the well-known sequence $1, 1, 2, 6, 21, 112, 853, \ldots$. But because we can also permute the Hopf-linked rings on the given ring, the number of non-homeomorphic structures of linkage of N rings is bigger than the number of connected graphs on N vertices.

Motivation

- String theory + Loop quantum gravity + Causal set approach
- Continuity: wrong concept
- Finitism

Basic gravitating object

We could look at our construction as a lattice filed theory, where the basic object is a "gravitating" ring:

Figure: Gravitating ring embedded to $\mathbb{R}^3$

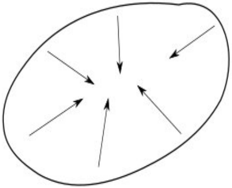
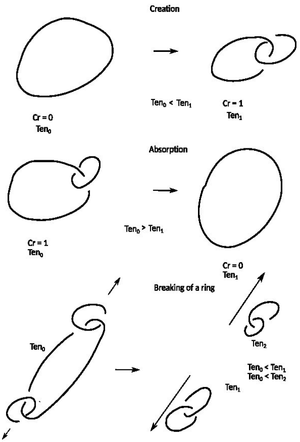


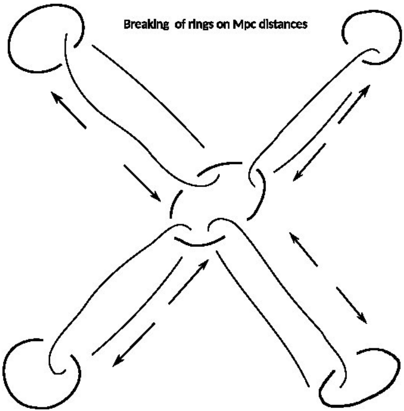
Figure: The basic processes with rings are creation, absorption and breaking of a ring.



The minimal length of these rings is 2 Planck lengths, but we can stretch it to multiples of Planck length. We define these 3 rules for **creation of a ring, absorption of a ring and finally the breaking of a ring.** The first diagram shows a creation of a ring on a ring with Ten_0 and $\text{Cr}_0 = 0$. The creation parameter of the original ring changes to $\text{Cr}_1 = 1$ and the tension parameter increases to $\text{Ten}_1 > \text{Ten}_0$. The opposite process is an absorption of a ring by other ring on the second diagram. The creation parameter changes from $\text{Cr} = 1$ to $\text{Cr} = 0$ and tension parameter decreases, $\text{Ten}_0 > \text{Ten}_1$. The last process is a breaking of a ring and creation of 2 new rings. This is phenomenological characterization of the processes which could happen to the rings. The details will tell us the master equations.

We posed many questions in this theory, which we want to solve. But let's ask the most important question, whether our theory could explain something, what other approches to QG (like string theory or loop quantum gravity, do not solve. We want to indicate now that RT has the potential to give an explanation to the following two problems: the first problem is the existence of **dark energy** and the second one is the existence of **arrow of time** in our Universe; Of course, we need to give correct proofs, but we will now only suggest possible solutions of these problems in RT. Let's start, for example, with the dark energy problem.

Figure: When a big number of rings break on Mpc distances, the Universe starts to accelerate.



- We need later explain why is the Universe accelerating in the context of RT. We already know that there was one hypothetical epoch of accelerated expansion at the beginning of the history of the Universe, which is called **cosmological inflation**. The second epoch is the **late time cosmic acceleration**. Both epochs could be modeled by RT by the process of breaking of big number of rings, Figure 6. Rings could have certain discrete values of inner resistance for stretching. (They can break for certain values of parameter Ten.) When the big number of rings break, the Universe starts to accelerate. We will need to obtain mathematical details of this process.
- The other problem, which we want to explain by RT, is the **problem of arrow of time in our Universe**. Our "local" void is traveling along some ring according to RT. This ring(s) gave the birth to time in our part of the Universe, because it "induced" in it the orientation.

Figure: Linkage of 1, 2 and 3 rings, which are hooked on some ring.

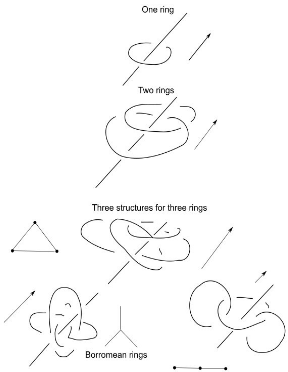


Figure: One branch of linkage of 4 and 5 rings (5 rings with 4 and 5 edges), which are hooked on some ring, when we represent intersection of rings by graphs. They should represent 8 gluons in the correspondence to the elementary particle table of the standard model.

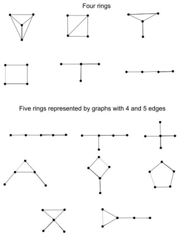
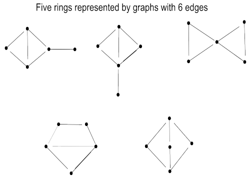


Figure: 5 rings with 6 edges. They should represent W^+, W^- and Z bosons, and the Higgs boson. There are also other graphs for 5 vertices with more then 6 edges. But it is an interesting thing that this table and the previous ones look like the elementary particle table of the standard model. (The coincidence is almost exact, but there are 2 structrues for Higgs boson.) So it would be necessary to show that there is some connection between irreducible representations of Poincarè group and the topological properties of linkage of rings in the space $\mathbb{R}^3$.



One mathematical problem in quantum gravity

J.Novák*

(Dated: June 12, 2018)

We formulate one mathematical problem from combinatorical topology. We show a possible application to quantum gravity and we discuss the connections with other theories.

* Physics Department, Technical University in Liberec, Studentská 1402/2, 461 17, Liberec, Czech republic

Quantum Gravity as theory of rings

Jan Novák*[1]

[1]Department of physics, Technical University in Liberec, Studentská 1402/2, 461 17, Liberec, Czech republic

May 9, 2018

Everything is made of atoms.

Richard P.Feynman

Abstract. The long-standing problem of general relativity is that is formulated in spacetime, which is continuous. We argument for discretization of space to loops, which we call rings. We define basic processes for these rings, which could be considered like laws of quantum gravity. We present the basic problems, which can be solved inside this theory. There is formulated one interesting mathematical problem from topology at the end.

Keywords. causal set approach, quantum causal histories, string theory, loop quantum gravity, dark matter, dark energy, arrow of time, non-locality, background independence, twistor theory

Introduction

It is a well-known fact that general relativity (GR) is formulated in spacetime, which is continuous. Many current theories lead to the idea of discretization of spacetime, because infinities of GR and quantum field theory (QFT) are caused by lack of short distance cut-off in degrees of freedom. Renormalization procedures helps in quantum field theories, but returns in naive attempts to quantize gravity.

Our approach is to discretize space to loops, which we call rings. We define basic processes for these rings, which are creation of a ring, absorption of a ring and breaking of a ring. Then we show, how we measure distances in this theory. Finally we want to solve like an application of this construction the problem of dark energy and arrow of time. There is formulated one interesting problem from combinatorical topology at the end. This article must be read as a work in progress.

*jan.novak@johnynewman.com

johnynewman.com

Sources of pictures: Physics Libre Texts, Hiroshima University, Science Blogs, University of Alberta, Einstein Online, Wordpress.com, LISA, Updatepedia, Wikimedia Commons, Discrete causal theory (Benjamin Dribus)