Causal set approach to quantum gravity and one mathematical problem

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Quantum Universe

1. Higher level structure (quantum)
2. Lower level structure (classical)

- Not distinguish between on-shell and off-shell.
- Generalized Hasse diagrams.
Causal metric hypothesis

The properties of the physical Universe are manifestations of causal structure.
Causal metric hypothesis

The causal structure of relativistic spacetime determines its metric structure up to scale:

- Rafael Sorkin: order plus number equals geometry
- Stephen Hawking: topological structure determines conformal structure
- David Malament: causal structure determines topological structure

 topological, smooth, causal, conformal, metric
Causal set approach

- Describing fundamental spacetime structure.
- Modeling gravitation at the quantum level.
- Unifying physical laws.

One of the results: heuristic bound on the value of cosmological constant in concordance with the experiment
Let $\tau : D_i \to D_t$ be a transition of directed sets, and let $Aut(D_i)$ and $Aut(D_t)$ be the automorphism groups of the source $D_i$ and the target $D_t$ of $\tau$, respectively. Let $F_\tau \equiv Aut(D_t ; \tau(D_i))$ be the subgroup of $Aut(D_t)$ whose elements permute $\tau(D_i)$ and its complement separately.
Each element of $F_\tau$ defines an automorphism of $D_i$, via restriction and conjugation by $\tau$. We refer to $F_\tau$ as the group of extensions of symmetries of $D_i$. There is a group homomorphism

$$\rho_\tau : F_\tau \to Aut(D_i),$$

$$\beta \mapsto \tau^{-1} \circ \beta \circ \tau,$$

which we refer to as the restriction homomorphism, where the factors $\tau^{-1}$ and $\beta$ in the composition are understood to be restricted to the image $\tau(D_i)$ of $\tau$. 
If $\rho_\tau$ is surjective, then $\tau$ **preserves the symmetries** of the source $D_i$ of $\tau$. The kernel $K_\tau$ of $\rho_\tau$ is the normal subgroup of $F_\tau$ whose elements consist of automorphism of $D_t$ fixing $\tau(D_i)$.

$$1 \rightarrow K_\tau \overset{i}{\rightarrow} F_\tau \overset{\rho_\tau}{\rightarrow} Aut(D_i) \rightarrow 1,$$

where $i$ is the inclusion of the kernel $K_\tau$ of $\rho_\tau$ into $Aut(D_i)$. Hence $F_\tau$ is a group extension of $Aut(D_i)$ by $K_\tau$ if and only if $\tau$ preserves the symmetries of $D_i$. 
The quotient group $G_\tau \equiv F_\tau / K_\tau$ may be identified with the normal subgroup of $F_\tau$ fixing the complement of $\tau(D_i)$ in $D_t$. The group $F_\tau$ is isomorphic to the group direct product of $K_\tau$ and $G_\tau$:

$$F_\tau \cong K_\tau \times G_\tau$$
Whether or not $\rho_\tau$ is surjective, it induces a quotient monomorphism $\bar{\rho}_\tau : G_\tau \to Aut(D_i)$, identifying $G_\tau$ with the subgroup of automorphisms of $D_i$ preserved by $\tau$. In the finite case, the subgroup index $[F_\tau : G_\tau]$ of $G_\tau$ in $F_\tau$ measures the number of different ways in which automorphism of $D_i$ preserved by $\tau$ are extended by $\tau$. If $\rho_\tau$ is surjective, then $\bar{\rho}_\tau$ is an isomorphism. If $\rho_\tau$ is not surjective, then every element of the complement $Aut(D_i) - \bar{\rho}_\tau(G_\tau)$ represents a symmetry broken by $\tau$. In the finite case, the subgroup index $[Aut(D_i) : \bar{\rho}_\tau(G_\tau)]$ measures the extent of this symmetry breaking. The groups $K_\tau$ and $G_\tau$ may be called causal Galois groups, since they are groups of of automorphisms fixing distinguished subobjects of directed sets in the context of discrete causal theory.
Continuum based theories

1. Divergence issues
2. Lack of natural scale
3. Experimental discreteness
4. Discreteness arising from continuum based assumptions
5. Discreteness via the philosophy of measurement
We formulate one mathematical problem, which will be useful further. We will call, that a circle $S^1 \in \mathbb{R}^3$ with finite length and finite circumference, which could be deformed, is a ring: let’s have a finite collection of $N$ rings $S^1$ in $\mathbb{R}^3$, which could not touch. Derive a formula for number of non-homeomorphic structures, which could be constructed from this finite collection of rings; Every two rings could be linked only once, they could not be knotted or twisted.
Partial solution of the problem

We have immediately one bound from below on the **number of non-homeomorphic structures**, when we map the linkage of rings to finite connected graphs on $N$ vertices. We simply exchange two rings, which are Hopf-linked by two vertices connected by an edge. So the number of linkage of $N$ rings is at least so big as the number of connected graphs on $N$ vertices. This is the well-known sequence 1, 1, 2, 6, 21, 112, 853, .... But because we can also permute the Hopf-linked rings on the given ring, the number of non-homeomorphic structures of linkage of $N$ rings is bigger than the number of connected graphs on $N$ vertices.
Motivation

- String theory + Loop quantum gravity + Causal set approach
- Continuity: wrong concept
- Finitism
Basic gravitating object

We could look at our construction as a lattice field theory, where the basic object is a "gravitating" ring:

**Figure:** Gravitating ring embedded to $\mathbb{R}^3$
**Figure:** The basic processes with rings are creation, absorption and breaking of a ring.
The minimal length of these rings is 2 Planck lengths, but we can stretch it to multiples of Planck length. We define these 3 rules for **creation of a ring, absorption of a ring and finally the breaking of a ring**. The first diagram shows a creation of a ring on a ring with $\text{Ten}_0$ and $\text{Cr}_0 = 0$. The creation parameter of the original ring changes to $\text{Cr}_1 = 1$ and the tension parameter increases to $\text{Ten}_1 > \text{Ten}_0$. The opposite process is an absorption of a ring by other ring on the second diagram. The creation parameter changes from $\text{Cr} = 1$ to $\text{Cr} = 0$ and tension parameter decreases, $\text{Ten}_0 > \text{Ten}_1$. The last process is a breaking of a ring and creation of 2 new rings. This is phenomenological characterization of the processes which could happen to the rings. The details will tell us the master equations.
We posed many questions in this theory, which we want to solve. But let’s ask the most important question, whether our theory could explain something what other approaches to QG like string theory or loop quantum gravity do not solve. We want to indicate now that RT has the potential to give an explanation to the following two problems: the first problem is the existence of 

**dark energy** and the second one is the existence of 

**arrow of time** in our Universe; Of course, we need to give correct proofs, but we will now only suggest possible solutions of these problems in RT. Let’s start, for example, with the dark energy problem.
**Figure:** When a big number of rings break on Mpc distances, the Universe starts to accelerate.
We need later explain why is the Universe accelerating in the context of RT. We already know that there was one hypothetical epoch of accelerated expansion at the beginning of the history of the Universe, which is called *cosmological inflation*. The second epoch is the *late time cosmic acceleration*. Both epochs could be modeled by RT by the process of breaking of big number of rings, Figure 6. Rings could have certain discrete values of inner resistance for stretching. (They can break for certain values of parameter Ten.) When the big number of rings break, the Universe starts to accelerate. We will need to obtain mathematical details of this process.

The other problem, which we want to explain by RT, is the *problem of arrow of time in our Universe*. Our "local" void is traveling along some ring according to RT. This ring(s) gave the birth to time in our part of the Universe, because it "induced" in it the orientation.
Figure: Linkage of 1, 2 and 3 rings, which are hooked on some ring.
Figure: One branch of linkage of 4 and 5 rings (5 rings with 4 and 5 edges), which are hooked on some ring, when we represent intersection of rings by graphs. They should represent 8 gluons in the correspondence to the elementary particle table of the standard model.
**Figure:** 5 rings with 6 edges. They should represent $W^+$, $W^-$ and $Z$ bosons, and the Higgs boson. There are also other graphs for 5 vertices with more than 6 edges. But it is an interesting thing that this table and the previous ones look like the elementary particle table of the standard model. (The coincidence is almost exact, but there are 2 structures for Higgs boson.) So it would be necessary to show that there is some connection between irreducible representations of Poincarè group and the topological properties of linkage of rings in the space $\mathbb{R}^3$. 

![Five rings represented by graphs with 6 edges](image.png)
One mathematical problem in quantum gravity

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We formulate one mathematical problem from combinatorial topology. We show a possible application to quantum gravity and we discuss the connections with other theories.

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Quantum Gravity as theory of rings

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Everything is made of atoms.

Richard P. Feynman

Abstract. The long-standing problem of general relativity is that it is formulated in spacetime, which is continuous. We argue for discretization of space to loops, which we call rings. We define basic processes for these rings, which could be considered like laws of quantum gravity. We present the basic problems, which can be solved inside this theory. There is formulated one interesting mathematical problem from topology at the end.

Keywords. causal set approach, quantum causal histories, string theory, loop quantum gravity, dark matter, dark energy, arrow of time, non-locality, background independence, twistor theory

Introduction

It is a well-known fact that general relativity (GR) is formulated in spacetime, which is continuous. Many current theories lead to the idea of discretization of spacetime, because infinities of GR and quantum field theory (QFT) are caused by lack of short distance cut-off in degrees of freedom. Renormalization procedures helps in quantum field theories, but returns in naive attempts to quantize gravity.

Our approach is to discretize space to loops, which we call rings. We define basic processes for these rings, which are creation of a ring, absorption of a ring and breaking of a ring. Then we show, how we measure distances in this theory. Finally we want to solve like an application of this construction the problem of dark energy and arrow of time. There is formulated one interesting problem from combinatorical topology at the end. This article must be read as a work in progress.

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Thank You!

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Source of picture: Discrete causal theory (Benjamin Dribus)