Issues in discrete formalisms in Quantum Gravity

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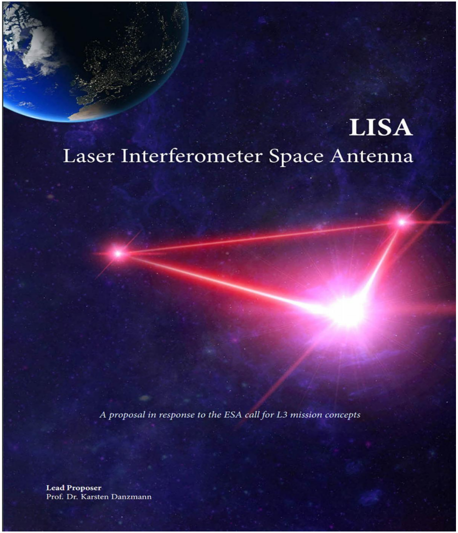
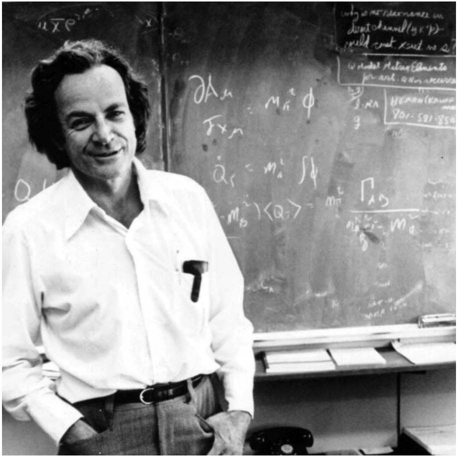


It doesn’t matter how beautiful your theory is, it doesn’t matter how smart you are or what your name is. If it doesn’t agree with an experiment, it is wrong.



- No evidence about any phenomenon in QG regime.
- Common features for all approaches to QG: String theory (ST), Loop Quantum Gravity (LQG), Causal set approach (CSA)

Nonlocality

CSA is based on five basic axioms:

1. Binary axiom
2. Measure axiom
3. Countability axiom
4. Star finiteness
5. Irreflexivity

Dark energy in CSA

Exercise: let us assume that the value about which Λ fluctuates is strictly zero; $\langle\Lambda\rangle = 0$. A rough estimate identifying spacetime volume with the Hubble scale H^{-1} then yields

$$V \sim (H^{-1})^4 \sim H^{-4} \to \Lambda \sim V^{-\frac{1}{2}} \sim H^2 \sim \rho_{critical}.$$

Exercise in non-locality

Suppose the extent of the non-locality is usually of the order $\rho \sim \frac{1}{l^4}$. When $\rho << \frac{1}{l^4}$, we would deal with a long range non-locality. The expression for $B_\rho^{(2)}$ was already derived and it has the following form

$$\rho^{-1}(B_\rho^{(2)}\Phi)(x) = a^{(2)}\Phi(x) + \sum_{n=0}^{2} b_n^{(2)} \sum_{y \in I_n(x)} \Phi(y),$$

$a^{(2)} = -2,\ b^{(0)} = 4,\ b^{(1)} = -8,\ b^{(2)} = 4$ and I_n means a distance of exactly n chronons on the causal set.

Quantization of gravity means quantization of geometry. There is defined a hierarchy of structures: **point set events, topological structure, differentiable manifold, causal structure, Lorentzian structure**; What structures should be quantized? To which structures should we apply the superposition principle. Structures, which are not quantized remain absolute.

Problem of time

Particular problem in the BI is the problem of time. We know that time is an absolute parameter in quantum mechanics. We can see it explicitely in Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi,$$

where ψ is the wave function and H the Hamiltonian.

Dimensional reduction

- evidence in ST, CSA, CDT, also LQG
- leads to scale invariant spectrum of cosmological perturbations even without CI

Determinism

Many of hidden variables theories rejected in the past for main two reasons:

1. counter examples could be constructed using eigenstates of certain symmetries: rotation sym., isospin sym., etc.
2. no need for such theories in the past

But at the Planck scale are the most of the familiar symmetries absent. Constructing counter examples to hidden variable theories is then harder.

1. Einstein's wish for reality
2. Quantum Cosmology
3. Even at a local scale there are troubles with quantum mechanics: these are, for example
    - the non-renormalizability of gravity
    - black holes
    - cosmological constant problem

Let's work in the Heisenberg representation in quantum mechanics now. If a complete set of operators $O(t)$ can be found that mutually commute at all times

$$[O(t), O(t')] = 0, \forall (t, t'), \tag{1}$$

then the theory may be set to be deterministic.

Feynman path integral

- different approaches to Quantum Mechanics (QM) during 20th century
- one of the most successful reformulations is due to Richard Feynman, so called path integral

$$U(x_b, t_b; x_a, t_a) = \int_{x(t_a)=x_a}^{x(t_b)=x_b} e^{\mathrm{i}S[x(t)]/\hbar} Dx(t).$$

- the problematic step is in the mathematical formulation the division of the interval and the limiting process

EPR experiment

- our paradigm, which we will formulate at the end, have immediate consequences for a well known experiment: Einstein, Podolski, Rosen experiment (EPR)
- a particle of vanishing spin decays in EPR experiment to two particles with spin 1/2
- after measuring the spin of one particle, the spin of the second one is determined; but how this would be possible when there is no communication between these two particles?

Wheeler delayed choice experiment

- Young's double slit experiment
- the striking feature is the phenomenon that a wave travelling simultaneously both ways is incompatible with a particle like behaviour
- Wheeler delayed choice experiment is realized like Young's double slit experiment but on cosmic distances.

Arrow of time

- we want to study for a while what is the difference between past and future in physics
- the concept of entropy and connections to the second law of thermodynamics
- the hard part is why do we find systems in low-entropy states at the beginning if these states are so unlikely

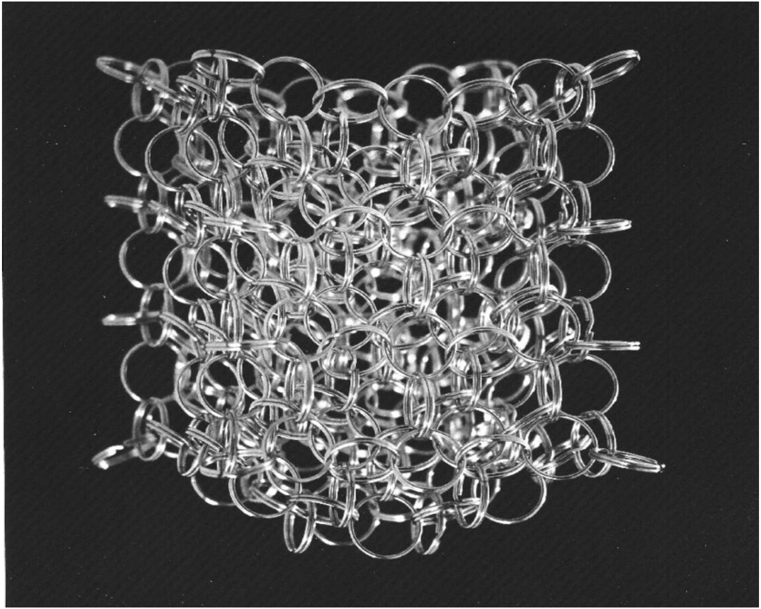
Dark matter

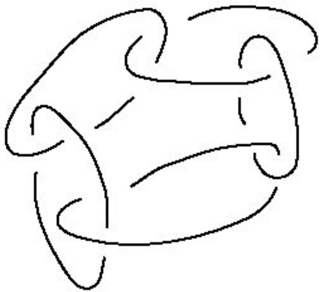
- we know today only 5% from the **energetical content** of the Universe and this state of knowledge is definitely not satisfactory. Approximately 25% creates, so called, **dark matter (DM)** and 70% creates dark energy
- we have many candidates: WIMPS, neutrinos, SUSY particles, and many others

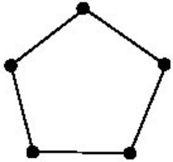
Mathematical apparatus

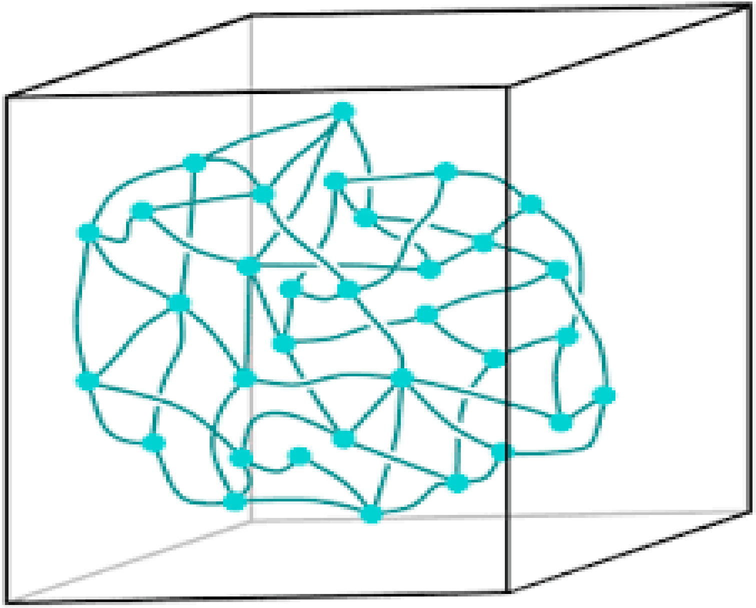
We will call, that a circle $S^1 \subset \mathbb{R}^3$ with finite length and finite circumference (we have a picture of torus in our mind), which could be deformed, is a **ring.**

Mathematical problem: we have a finite collection of N rings S^1 in $\mathbb{R}^3$, which could not touch; Derive a formula for number of non-homeomorphic structures, which could be constructed from this finite collection of rings. Every two rings could be linked only once, they could not be knotted or twisted.









We have immediately one bound from below **on the number of non-homeomorphic structures**, when we map the linkage of rings to finite connected graphs on N vertices. We simply exchange two rings, which are Hopf-linked **by two vertices connected by an edge.** So the number of linkage of N rings is at least so big as the number of connected graphs on N vertices. This is the well-known sequence $1, 1, 2, 6, 21, 112, 853, ...$

Definition

A decorated permutation of the set $[n]$ is a bijection $\pi : [n] \to [n]$ whose fixed points are colored either black or white. We denote a black fixed point $\pi(i) = \underline{i}$ and white fixed point by $\pi(i) = \bar{i}$. An antiexcendance of the decorated permutation π is an element $i \in [n]$ such that either $\pi^{-1}(i) > i$ or $\pi(i) = \bar{i}$ (i is a white fixed point).

Definition

Fix k and n. Given a partition λ, we let Y_λ denote the Young diagram associated to λ. A Le-diagram D of shape λ and type (k, n) is a Young diagram of shape Y_λ contained in a $k \times (n-k)$ rectangle, whose boxes are filled with 0 and 1 in such a way that the *Le*-property is satisfied: there is no 0 which has 1 above it in the same column and a 1 to its left in the same row.

Definition

A plabic graph is an undirected planar graph G drawn inside a disk (considered modulo homotopy) with n boundary vertices on the boundary of the disk, labeled $1, \ldots, n$ in clockwise order, as well as some colored internal vertices. These internal vertices are strictly inside the disk and are each colored either black or white. Morever, each boundary vertex i in G is incident to a single edge. If a boundary vertex is adjacent to a leaf (vertex of degree 1), we refer to that leaf as a lollipop.

Definition

A perfect orientation O of a plabic graph G is a choice of orientation of each of its edges such that each black internal vertex u is incident to exactly one edge directed away from u, and each white internal vertex v is incident to exactly one edge directed towards v. A plabic graph is called perfectly orientable if it admits a perfect orientation. Let G_o denote denote the directed graph associated with a perfect orientation O of G. Since each boundary vertex is incident to a single edge it is either source (if it is incident to an outgoing edge) or a sink (if it is incident to an incoming edge) in G_o. The souce set $I_0 \subset [n]$ is the set of boundary vertices, which are sources in G_0.

We want to conclude that we need to find an apparatus, how to work with the **RT paradigm**, which we will introduce now. These plabic graphs will enable it to us.

We want to show that there is hidden a philosophical concept that could lead to a new theory in the physical foundations of ST, LQG and CSA to quantum gravity. We will use the knowledge that particles are not pointlike objects. But the spacetime will be **not continous** for us. So, ST will be for us just a toy model.

We all know very well that general relativity is pertubatively non-renormalizable. This means that when we try to construct **Feynman diagrams** and deal with **gravitons** similarly as in quantum mechanics the theory diverges.

Another fact from a different area of physics is that the **notion of particles** is non-unique in quantum field theory in curved background. This serves us as an inspiration for our construction of graviton on fundamentally nonlinear level.

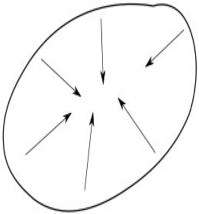
The first thing what we need to do is to prove that our discretization is correct according to a deep principle, the so called **holographical principle**. It states that the area of any surface S enclosing a volume V measures the information content of the underlying theory in the volume V. Our discretization is in concordence with this principle. The number of rings scales as the area of the enclosed volume.

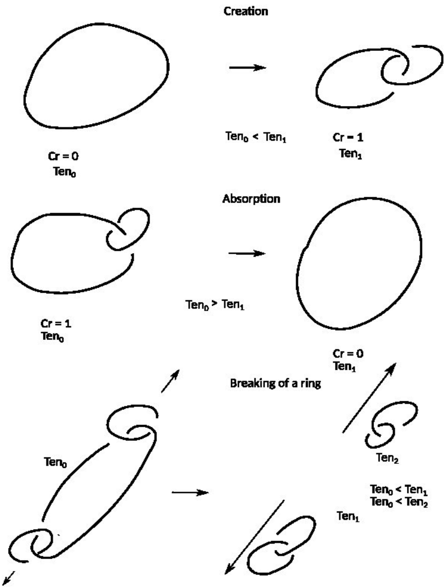
An urgent question comes to our mind. Is this discretization just a mathematical tool or a real **physical object**? We claim that it is not a mathematical abstraction.

A keynote about nonlinear graviton

- Could we see just one graviton in an apparatus?
- Too tiny effect, but we could not see it principially.

Gravitating ring

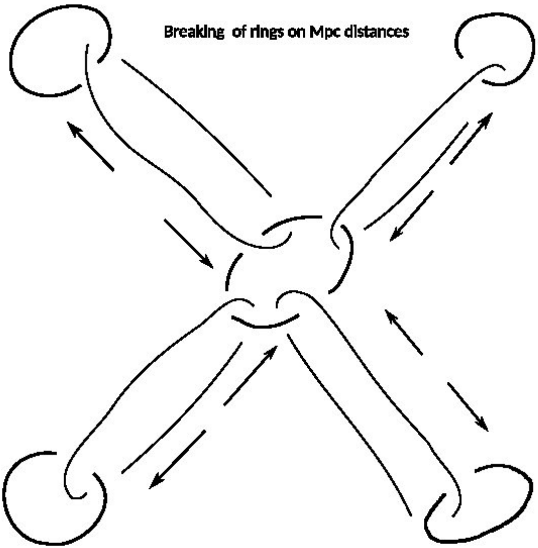


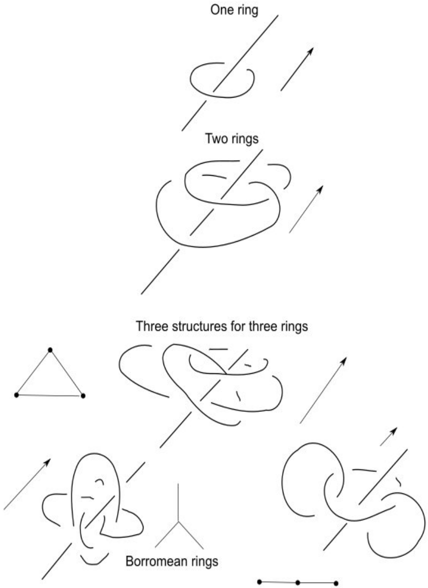

Creation
Cr = 0
Ten_0
$Ten_0 < Ten_1$
Cr = 1
Ten_1
Absorption
Cr = 1
Ten_0
$Ten_0 > Ten_1$
Cr = 0
Ten_1
Breaking of a ring
Ten_0
Ten_2
$Ten_0 < Ten_1$
$Ten_0 < Ten_2$
Ten_1

RT-paradigm

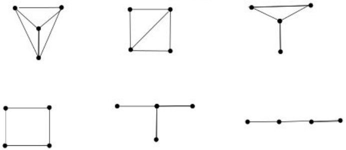
How we will solve the basic common problems:

- Dark energy
- Dimensional reduction
- Feynman path integral
- Determinism
- Dark matter
- Arrow of time
- Wheeler delayed choice experiment
- EPR paradox
- Nonlocality
- Background independence

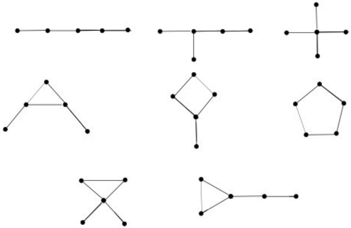

Breaking of rings on Mpc distances


One ring
Two rings
Three structures for three rings
Borromean rings

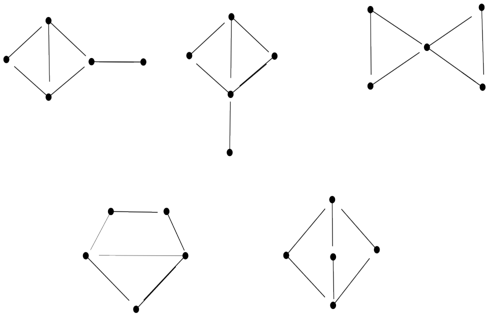
Four rings



Five rings represented by graphs with 4 and 5 edges

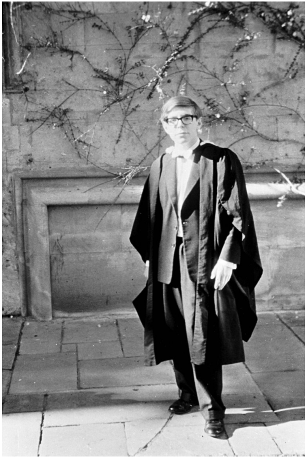


Five rings represented by graphs with 6 edges



- We claim that our paradigm could be made background independent. But general relativity is only partially background independent. Is not the RT paradigm also only partially background independent?
- What is the origin of the first ring? Could it mean that we really should prefer ekpyrotic-type of models in cosmology?
- Gravitational waves

S.W.Hawking: Remember to look up to the stars and not down at Your feet.



Thank You!

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