Adventures of modern cosmology

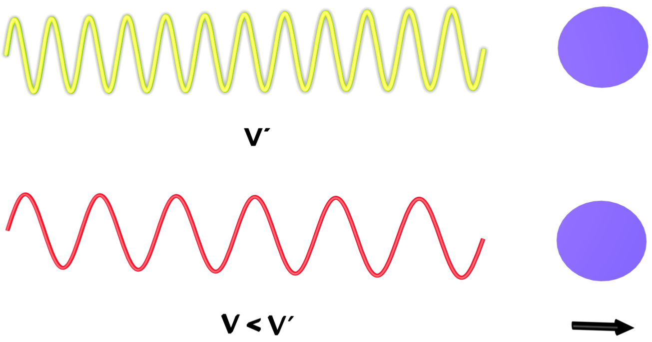
Jan Novák

previous employment at Technical University of Liberec

Matej Bel University, 17.1.2022

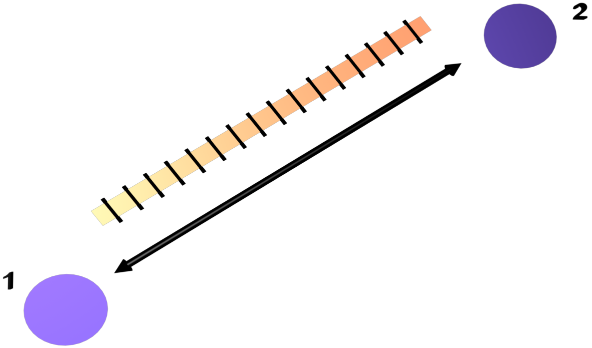
Hubble's discovery

We could find the origin of modern cosmology in the 1920s, when E. Hubble made the amazing discovery that the Universe is **expanding**.



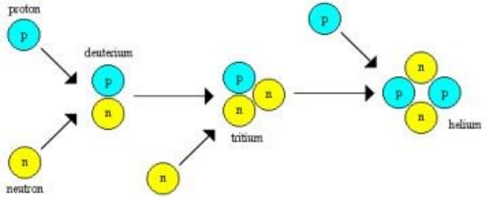
Scale factor

When the space is stretching or contracting, the gauge on the coordinate system remains firmly attached to it. The **scale factor** *a* measures the rate of expansion or contraction.



Big-Bang Nucleosynthesis

Observation of G. Gamow in 1946:



$$n + p \rightarrow D + \gamma$$

$$D + n \rightarrow {}^3H + \gamma$$

$$D + p \rightarrow {}^3He + \gamma$$

$$D + D \rightarrow {}^3H + p$$

$$D + D \rightarrow {}^3He + n$$

$$D + D \rightarrow {}^4He + \gamma$$

$${}^3H + p \rightarrow {}^4He + \gamma$$

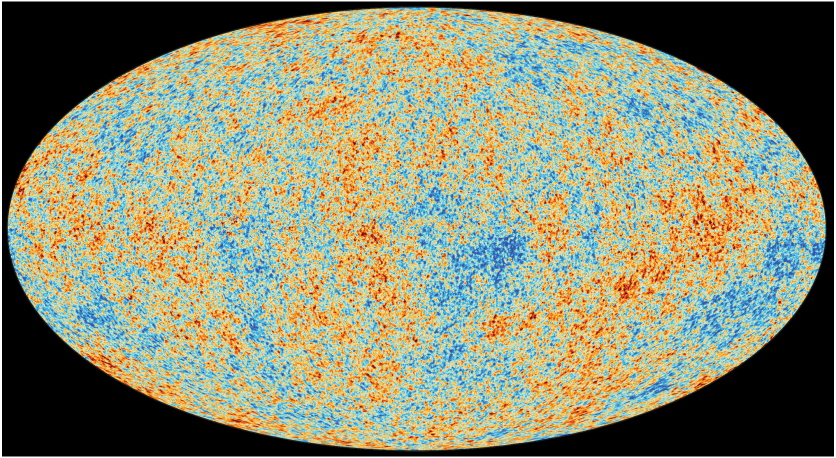
$${}^3He + n \rightarrow {}^4He + \gamma$$

$${}^3H + D \rightarrow {}^4He + n$$

$${}^3He + D \rightarrow {}^4He + p$$

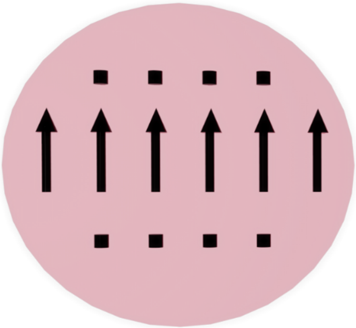
Cosmic microwave background

A. Penzias and R. Wilson found it in 1965. We know today that it has a temperature of $T_{CMB} = 2.725 \pm 0.002$ K.

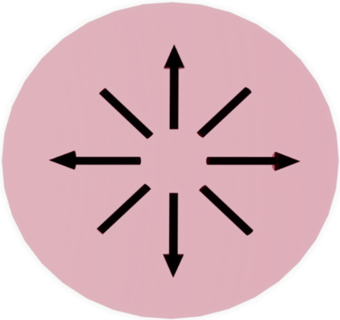


Simplification of the metric

We suppose that the Universe is **homogeneous and isotropic** on the biggest scales. Therefore, we could simplify the metric and compute the scale factor.



Homogeneous



Isotropic

The metric of a homogeneous and isotropic Universe has the following form in spherical coordinates:

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\theta + r^2 \sin^2\theta \, d\phi^2)$$

We plug this metric tensor into Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

where $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor, R is Ricci curvature and $T_{\mu\nu}$ is energy-momentum tensor.

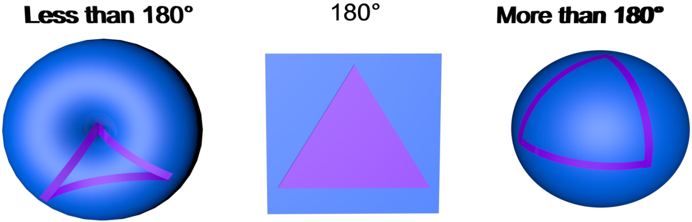
The only unknown function is the scale factor.

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p),$$

where ρ is energy density in the Universe, p is pressure and k is curvature, which could be rescaled in such a way that it takes the values $k = -1, 0, 1$.

We could have different types of **space geometry** in the Universe. The sum of all angles in a triangle could be **smaller** than 180 degrees, **equal** to 180 degrees, or **bigger** than 180 degrees.



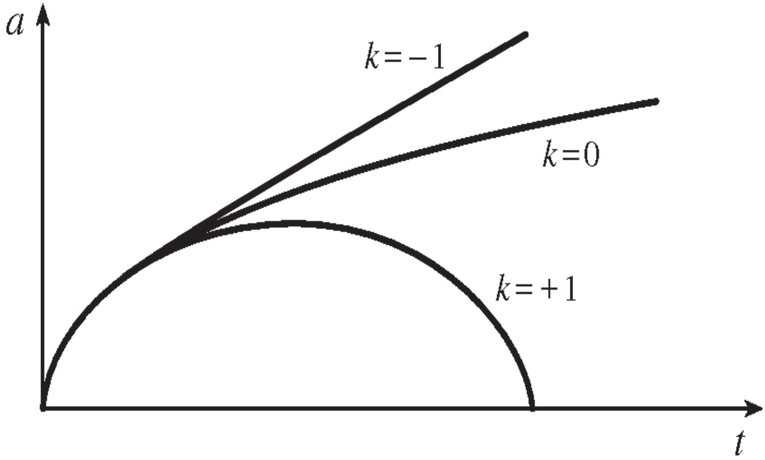
Continuity equation

We presented two of the three equations, which describe evolution of the scale factor $a = a(t)$ with time. The third equation is the **continuity equation**

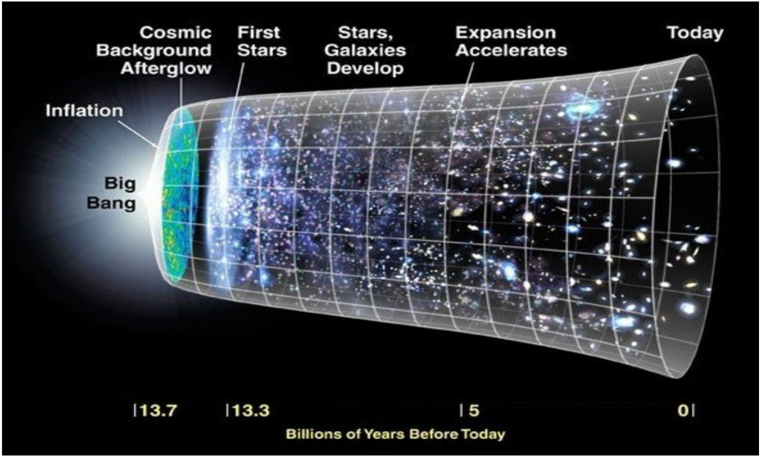
$$\dot{\rho} + 3H(\rho + p) = 0.$$

The Friedmann equations and the continuity equation are not independent. We could derive the third equation from the two Friedmann equations by algebraic manipulation and by derivatives.

When we solve the Friedmann equations for the 3 types of space geometry $(-1, 0, 1)$, we obtain Friedmann models. These models have only historical significance today.

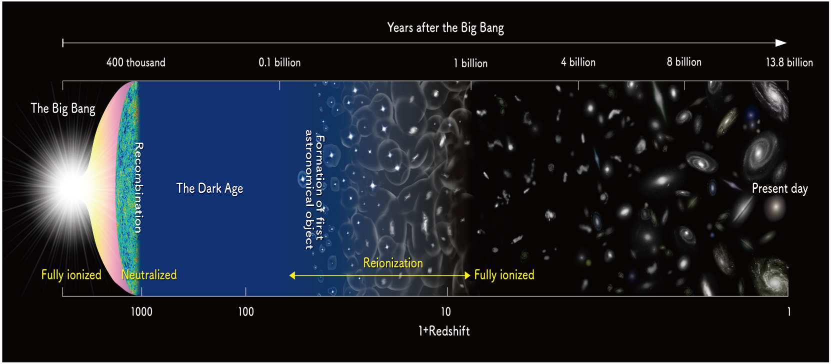


Hot Big Bang


Cosmic Background Afterglow
First Stars
Stars, Galaxies Develop
Expansion Accelerates
Today
Inflation
Big Bang
13.7
13.3
5
0
Billions of Years Before Today

Reionization

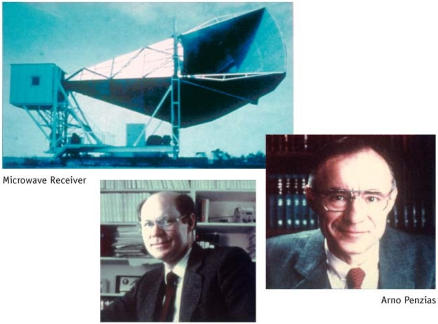
We know reionization took place when the Universe was approximately one **billion** years old, but we don't know exactly when and we don't know the precise effect it had on the earliest galaxies.



Cosmic microwave background

When we continue back in time, there had to be a moment in history when the density was so high that the Universe was **not transparent to electromagnetic radiation.** The time is approximately 13.8 billion years from today and 380 000 years after the Big Bang (BB).

DISCOVERY OF COSMIC BACKGROUND

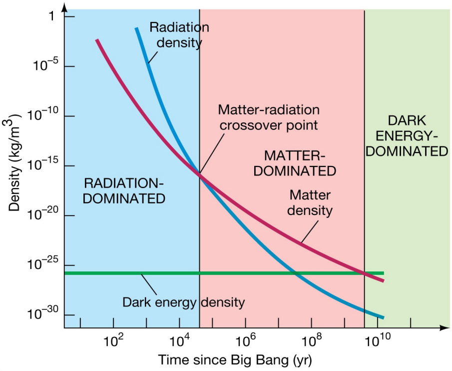


Microwave Receiver

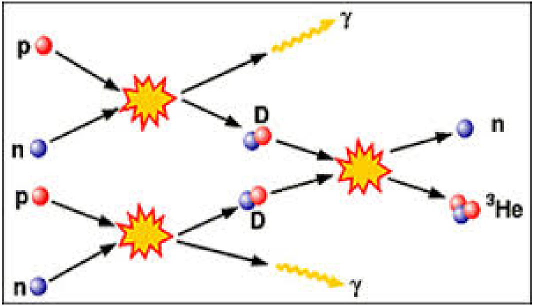
Arno Penzias

Robert Wilson

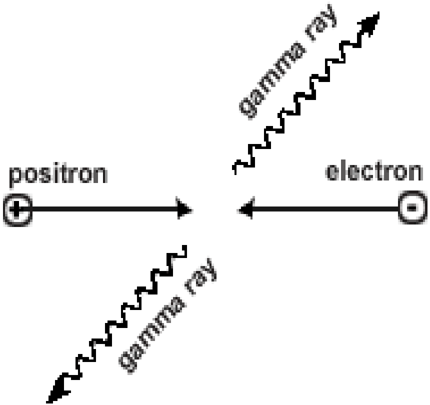
The **density of matter was roughly equal to the density of radiation** 50 000 years after the BB. Why did it have to be equal at a certain time after BB? We need to consider the equation of state for radiation and matter. $p_m = 0$ for matter and $p_r = \frac{\rho}{3}$ for radiation. When we plug into the continuity equation, we obtain the dependence $\rho_m \sim \frac{1}{a^3}$ for matter and $\rho_r \sim \frac{1}{a^4}$ for radiation.



Nuclear reactions started to be efficient approximately 200-300 s after the BB. Free protons and neutrons formed helium and other light elements at this time. **The abundance of light elements** is **in a very good agreement** what we see in the **Universe today**. When the Universe was younger, all nuclei, that were created were immediately destroyed by other protons and neutrons.

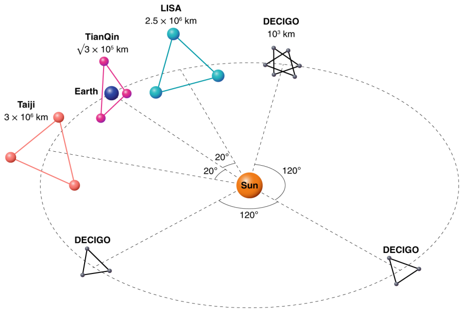


The typical energy at 1 second after the BB is of the order of electron mass (temperature 10^{10}K). The numerous **electron-positron** pairs began to **annihilate** when temperature drops below their rest mass. And only small excess of electrons over positrons, roughly one per billion photons, remains after annihilation.



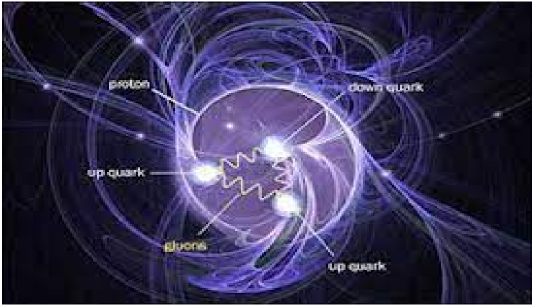
DECIGO and BBO

We have experimental verification, what was happening in the Universe after the time of 1 second. There is only one chance how to get experimental proofs about events in the Universe in shorter time than 1 s. This possibility gives us **gravitational waves**. The time will come when we will measure primordial gravitational wave background from the early Universe in the future.

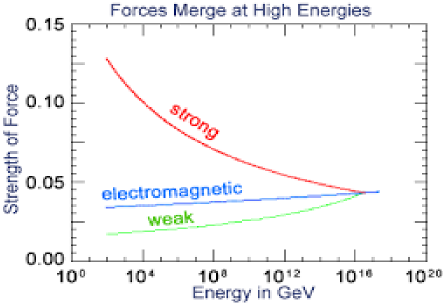


Two important events took place at the time 0.2 second after the BB, because certain **weak interaction processes fell out of equilibrium.**

The time of quark-gluon transition is 10^{-4} seconds after the BB. It means that **free quarks and gluons began to be confined in baryons and mesons**. The physics of this transition is not completely understood yet.



Electro-weak **symmetry breaking** took place at the time 10^{-10} second after the BB. The range for the energy scale for the interval $10^{-10} - 10^{-14}$ seconds is still reachable by our accelerators. The Standard model of electroweak and strong interactions appears to be applicable here. We finish at the time 10^{-36} second, which is the energetical scale of **Grand Unification.** It means that all three interactions (electro-magnetic, weak and strong) is unified into one interaction at this energy.



Quantum Gravity

When we want to do a unification of **GR** and **QFT**, we need to combine these three constants together: G, c and $\hbar$; The only possibility, how to obtain a quantity, which has a unit of time is the following combination

$$t_P = \sqrt{\frac{\hbar G}{c^5}}.$$

When we plug the values we obtain $t_P \sim 10^{-43}$s. This is roughly the time after the BB, when we need to start to use the QG (quantum gravity).

Problems of hot Big Bang theory

Does hot BB theory solve all main problems of cosmology? The answer is no. It does not solve the **problem of horizons**, **flatness problem** and **problem of monopoles**.

$$\rho_c \equiv \frac{3H^2}{8\pi G}.$$

We don't know exactly what is the current value of Hubble constant H_0. But when we insert this value of $H_0 = 73.8 \pm 2.4 \frac{\text{km}}{\text{s Mpc}}$ to the formula, the density of the Universe is very close to the critical density.

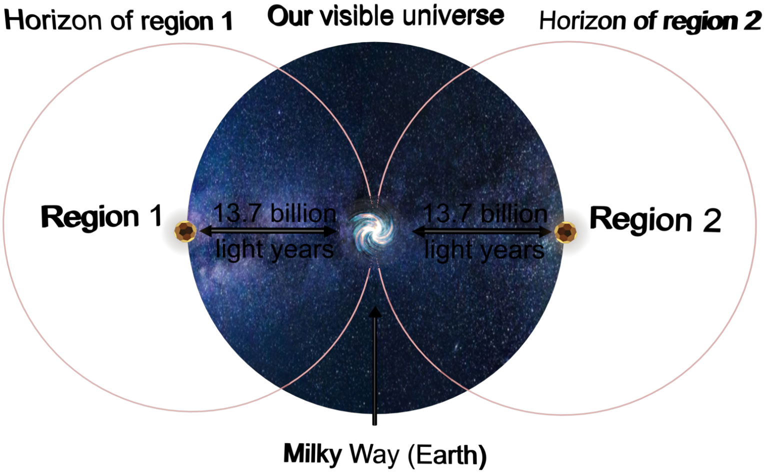
Let's denote $\Omega(t) \equiv \frac{\rho}{\rho_c}$. We can write the total density as $\Omega_{tot} = \Omega_0 + \Omega_\Lambda$, Ω_0 is the current density and Ω_Λ is the density coming from the cosmological constant term in Friedmann equation, $\Omega_\Lambda = \frac{\Lambda}{3H^2}$.

$$|\Omega_{tot}(t) - 1| = \frac{|k|}{a^2 H^2}.$$

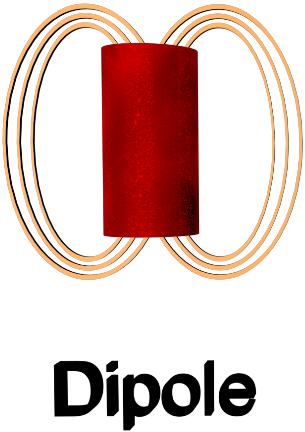
We suppose that normal matter is more dominant than cosmological constant or curvature term. Then we obtain that $a^2 H^2 \sim t^{-1}$ and $a^2 H^2 \sim t^{-2/3}$. So we have $|\Omega_{tot} - 1| \sim t$ and $|\Omega_{tot} - 1| \sim t^{2/3}$. It means that the difference between Ω_{tot} and 1 is an **increasing function of time.**

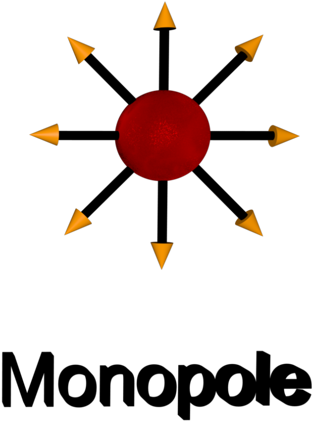
The Universe was extremely close to the flat space geometry at the beginning and it is necessary to explain this **fine-tunning.**

The problem of horizons refers to **communication between different regions** of the Universe.

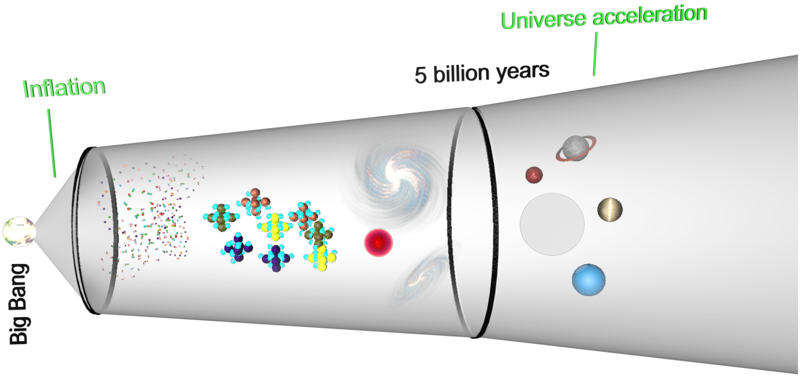


There should not exist **magnetic monopoles according to Maxwell equations**, but GUT theories predict their existence.





Cosmological inflation is solving all these three problems. It is a hypothetical rapid expansion at the beginning of the **universe in time** $10^{-43} - 10^{-32}$ **second** after the Big Bang. It is characterized by a simple condition $\ddot{a} > 0$.



We plug to the second Friedmann equation now and we put the cosmological constant term in the density of matter and radiation:

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p)$$

Because $\ddot{a} > 0$ then $p < -\frac{\rho}{3}$. And this is a **very strange material with a negative pressure.** Cosmological constant corresponds to $p = -\rho$, so it fulfills this condition.

When we want to study the properties of the period of inflation, we assume the extreme conditions $p = -\rho$, which simplifies the analysis. By inspecting the continuity equation and the first Friedmann equation (we put $k = 0$ at this moment), we learn that during this phase

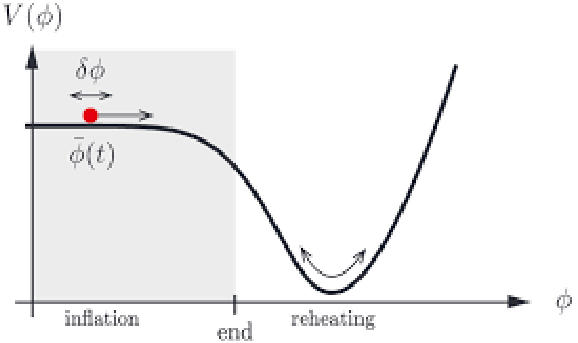
$$\rho = const.,\ H_i = const.,$$

where we have indicated by H_i **the value of the Hubble rate during inflation.** Correspondingly, we obtain

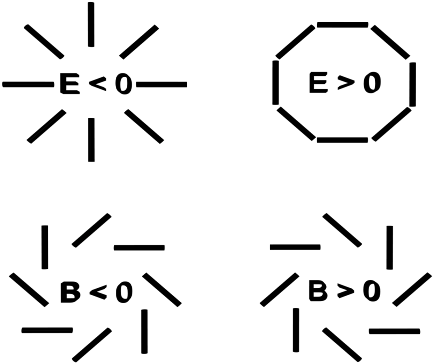
$$a = a_i e^{H_i(t-t_i)},$$

where t_i denotes the time at which inflation starts.

Let us conclude that cosmological inflation solves the three basic problems of hot Big Bang cosmology. If these problems would be all what is solved due to cosmological inflation, the theory would be not so successful as it is today. It was actually constructed for solving these three basic problems around 1981. It was found later that cosmological inflation solves also the **problem of primordial fluctuations** in early Universe, from which were later born the nuclei for galaxies.

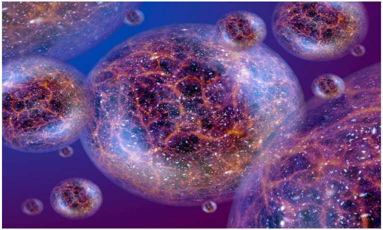


People already prepare other futuristic gravitational wave detectors as DECIGO or BBO. These experiments could measure also the **primordial gravitational wave background** from cosmological inflation. Simply, the rapid expansion of the Universe could leave an imprint in gravitational waves. The other possibility is to measure so-called **B-mode** in CMB.



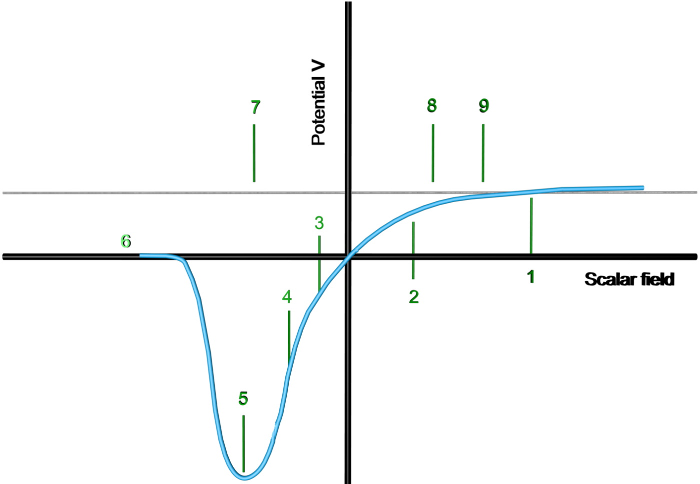
Multiverse

A. Vilenkin and others have shown that there is a possibility how to verify the hypothesis of multiverse. Imagine a picture of expanding disjoint bubbles. They could leave an imprint in our universe when these **bubbles of other universes collide** with our Universe. Again the observation of primordial gravitational wave background will be a key experimental result, which could give us an indication that something as multiverse could exist.



Cyclic models of the Universe

We will do a rough overview of one cycle, which should last approximately 1 trillion years.



The density of the scalar field and pressure is given followingly:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

When we plug to the Friedmann equations, where is only the scalar field as energy, we obtain

$$H^2 = \frac{8\pi G}{3}(\frac{1}{2}\dot{\phi}^2 + V(\phi)),$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}(\dot{\phi}^2 - V(\phi)).$$

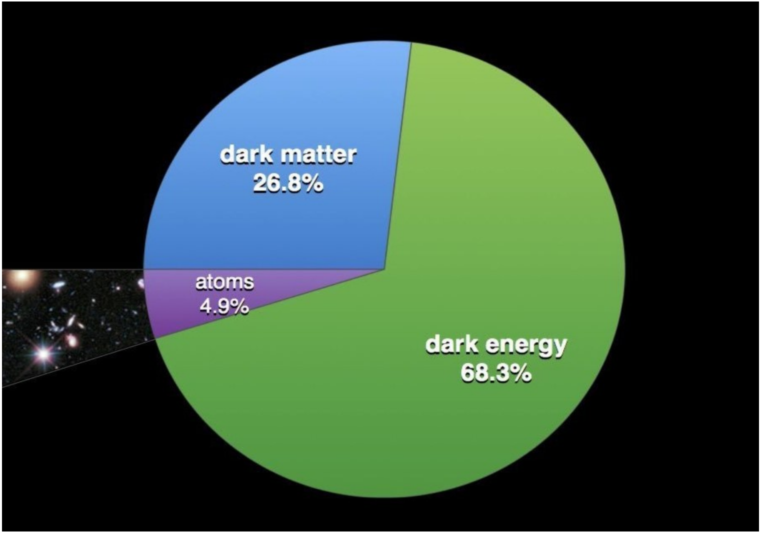
We need to look what will happen with the potential and kinetic term during the cycle.

Two faces of Johny Newman

Popularization book about physics with three articles:

- Adventures of modern cosmology
- Adventures of quantum gravity
- Are we alone?

Dark matter and dark energy



Classical explanation of the problem of dark energy

- add **cosmological constant** to Einstein equations
- introduce a new field: **scalar field** (phantom, quintom, quintessence)
- **modification of gravity**:
  1. modify the law of gravity at large distances
  2. build the models of gravity from higher-dimensional models

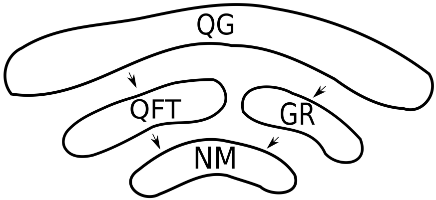
Example of modifications at large distances

The field equations for $f(R)$ theories are similar as field equations of general relativity:

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + \Box F g_{\mu\nu} - F_{;\mu;\nu} = 8\pi G T_{\mu\nu},$$

where $f(A)$ is an analytical function of A, $F(R) = \frac{df(R)}{dR}$, $R_{\mu\nu}$ is Ricci tensor, R is Ricci scalar, $T_{\mu\nu}$ is a tensor of energy momentum and $\Box F = \frac{\partial^2 F}{\partial t^2} - \frac{\partial^2 F}{\partial x^2} - \frac{\partial^2 F}{\partial y^2} - \frac{\partial^2 F}{\partial z^2}$. Unknown function is the metric function $g_{\mu\nu}$.

Quantum gravity and dark energy



1. string theory (origin from particle physics models)
2. loop quantum gravity (canonical quantization in general relativity)
3. causal set approach (built on 6 axioms)

String theory

1. Extra compact dimensions:

$$Q_1 \sim Q_2 \ \leftrightarrow x(Q_1) = x(Q_2) + 2\pi nR, \ n \in \mathbb{Z},$$

We could write without any confusions:

$$x \sim x + 2\pi R$$

which should be interpreted that we identify any two points that differ by $2\pi R$. The identification has changed the non-compact dimension to a compact one. We could look at the identifications as a complicated way how to think about the circle.

2. Conformal field theory: we increase symmetries in field theory
3. Supersymmetry: extra symmetry between bosons and fermions

Loop quantum gravity

Application of **LQG to cosmology**: Already one possible explanation gave us cyclic models of P. Steinhardt, but these are purely classical. We need to develop quantum models. LQG modified the Friedmann equation followingly:

$$H^2 = \frac{\dot{p}}{4p^2} = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_{crit}}\right),$$

where

$$\rho_{crit} = \frac{3}{8\pi GCp},$$

and C is a constant. It is clear that the expression H^2 changes sign when ρ reaches ρ_{crit}. It means that if we trace the evolution of the Universe backward, there is a point where H^2 vanishes.

Causal set approach

We suppose in quantum gravity that the relation

$$\Delta\Lambda\Delta V \sim \hbar$$

holds, where $\Delta\Lambda$ is a fluctuation in cosmological constant in given volume V.
The central result:

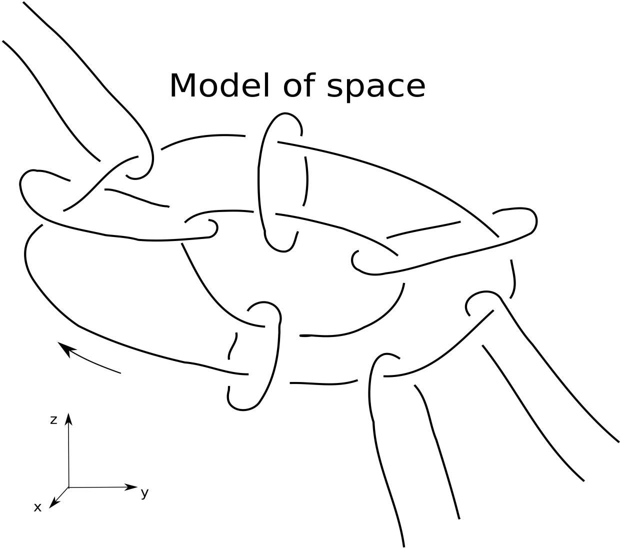
$$\Delta\Lambda \sim \frac{1}{\sqrt{V}}$$

The standard cosmological argument:

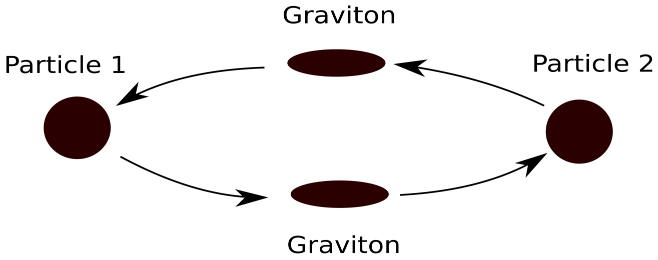
$$V \sim (H^{-1})^4 = H^{-4} \Rightarrow \Lambda \sim \frac{1}{\sqrt{V}} \sim H^2 \sim \rho_{crit}$$

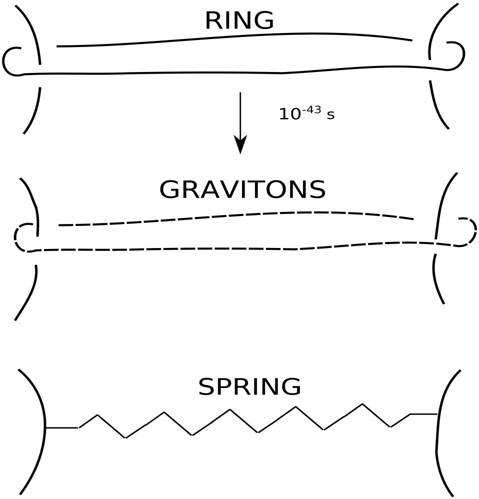
It implies that Λ will be everpresent at least in $3+1$ dimensions.

Graviton as a phonon

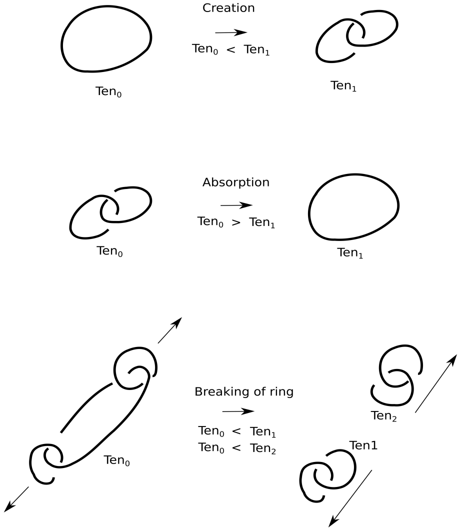


Old concept of interaction in gravity sector

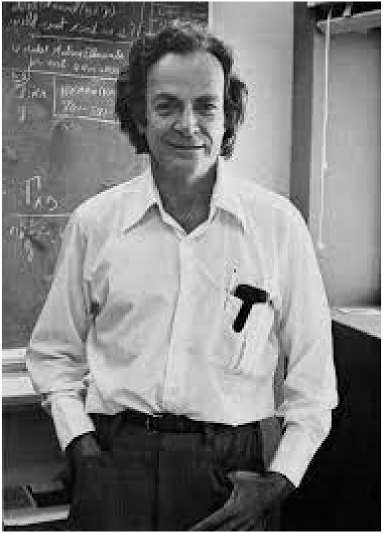



RING
10^{-43} s
GRAVITONS
SPRING

Model of dark energy in ring paradigm



If science is to progress, what we need is the ability to experiment.



Some pictures were taken from web and from popularization papers Adventures of modern cosmology or Adventures of quantum gravity.