Secrets of dark energy

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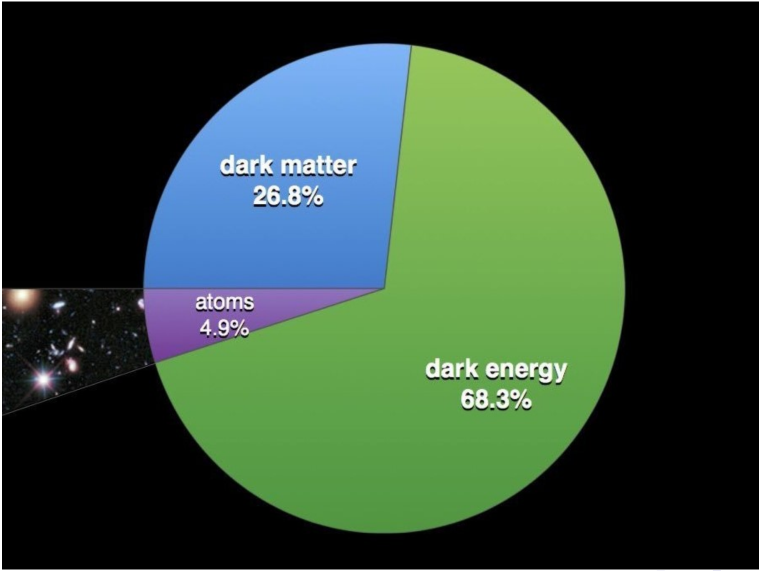
Dark energy



Classical explanation of the problem of the accelerated expansion

Two big groups of theories: dark energy models modify the stress-energy content by adding a component with equation of state $w \cong -1$ and the modified gravity category corresponds to changing the left-hand side of Einstein equations, which means that we correct the term in the Einstein-Hilbert action

- add **cosmological constant** to Einstein equations
- introduce a new field: **scalar field** (phantom, quintom, quintessence)
- **modification of gravity**:
  1. modify the law of gravity at large distances
  2. build the models of gravity from higher-dimensional models

f(R)-theories

Field equations:

$$F(R)R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T_{\mu\nu},$$
$$\mu, \nu = 0, 1, 2, 3$$

where $\kappa^2 = 8\pi G_N$ and $F(R) = f'(R)$. Besides,
$\Box F = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu F)$.
Trace of the equation:

$$3\Box F(R) + F(R)R - 2f(R) = \kappa^2 T,$$

$T = g^{\mu\nu}T_{\mu\nu}$.
Vacuum solutions for which Ricci scalar is constant $\rightarrow$ concept of de Sitter points:

$$RF(R) - 2f(R) = 0$$

DGP-models

$$S = \frac{M^3}{4}\int \sqrt{G}\mathscr{R}_{(5)}\, d^5X + \frac{M_P^2}{2}\int \sqrt{|g|}R\, d^4x,$$

where M is the fundamental five-dimensional Planck mass and M_P is the four-dimensional Planck mass. As they stand in this action M and M_P are independent. $G_{AB}(X) \equiv G_{AB}(x, y)$ denotes a 5D metric for which the 5D Ricci scalar is $\mathscr{R}_{(5)}$. The brane is located at $y = 0$. The induced metric on the brane is denoted by

$$g_{\mu\nu}(x) \equiv G_{\mu\nu}(x, y = 0)$$

The $4D$ Ricci scalar for $g_{\mu\nu}(x)$ is $R = R(x)$.

LDGP and QDGP models

- models with $w < -1$ but without quantum instability are an interesting possibility
- can occur in modified gravity theories, without any phantom matter
- the 4-dimensional brane universe contains only matter and cosmological constant Λ

The basic equations in LDGP models in cosmology look as following (L. P. Chimento, R. Lazkoz, R. Maartens, I. Quiros, arXiv:astro-ph/0605450):

$$\dot{\rho} + 3H\rho = 0,$$
$$H^2 + \frac{H}{r_c} = \frac{1}{3}(\rho + \Lambda),$$

where r_c is the crossover scale. At early times, for $r_c >> H$ we recover general relativistic Friedman equation, but at late times, the H/r_c term is important and the Friedman equation is nonstandard. Effective dark energy density is equal to

$$\rho_{eff} = \Lambda - 3\frac{H}{r_c}$$

and for the effective state parameter w_{eff} holds

$$1 + w_{eff} = \frac{\dot{H}}{r_c H \rho_{eff}}.$$

Because $\dot{H} < 0$, we have $w_{eff} < -1$. This is guaranteed by $\rho_{eff} > 0$ and the condition is fulfilled for $H < \frac{r_c \Lambda}{3}$. In general relativity with phantom matter, $w < -1$, $\dot{H}$ finally becomes positive and it leads to super-acceleration of the universe and Big Rip. But in our case is $\dot{H}$ always negative and we could avoid the **Big Rip singularity**. The LDGP models were further generalized to QDGP models, which allow the crossing of the phantom divide. Also in QDGP models is the Big Rip singularity avoided.

Quantum gravity and accelerated expansion

1. causal set approach (with cosmological constant)
2. discrete approaches to QG (with cosmological constant)
3. group field theory (with phantom matter)
4. metastring theory (with cosmological constant)
5. string gas cosmology (with phantom matter)
6. string cosmology based on AdS/CFT correpondence (with phantom matter)
7. theory based on non-commutative geometry (with new dark energy particle mitron)
8. asymptotic safety program (without dark energy)
9. entropic gravity (without dark energy)

Causal set approach

We suppose in quantum gravity that the relation

$$\Delta\Lambda\Delta V \sim \hbar$$

holds, where $\Delta\Lambda$ is a fluctuation in cosmological constant in given volume V.
The central result:

$$\Delta\Lambda \sim \frac{1}{\sqrt{V}}$$

The standard cosmological argument:

$$V \sim (H^{-1})^4 = H^{-4} \Rightarrow \Lambda \sim \frac{1}{\sqrt{V}} \sim H^2 \sim \rho_{crit}$$

It implies that Λ will be everpresent at least in $3+1$ dimensions (R. Sorkin, S. Surya, arXiv: 1903.11544).

Loop quantum gravity and other discrete approaches

- consider microscopic structure of spacetime and its interaction with matter
- discreteness of geometry and Lorentz invariance at low energies is a key aspect of QG
- massive fields are the natural candidates for probes of spacetime discreteness

The ’friction-like’ force must be proportional to **R** (**Ricci scalar**), mass m, **4-velocity** u^μ, **spin of the particle** s^μ and **time-like unit vector** ξ^μ specifying the local frame defined by the matter that curves spacetime. The formula is the following:

$$u^\mu \nabla_\mu u^\nu = \alpha \frac{m}{m_p^2} \mathrm{sign}(s \cdot \xi) \mathbf{R}\, s^\nu,$$

where α is a dimensionless coupling (A. Perez, D. Sudarsky, arXiv: 1711.05183).

We need to **break the general covariance** of the field equations of GR. However, people studied already a simple relaxation of general covariance from full coordinate invariance down to space-time volume preserving coordinate transformations. Such modification is called **unimodular gravity** and its field equations are just trace-free part of the Einstein field equations:

$$\mathbf{R}_{\mu\nu} - \frac{1}{4}\mathbf{R}g_{\mu\nu} = 8\pi G[\mathbf{T}_{\mu\nu} - \frac{1}{4}\mathbf{T}g_{\mu\nu}]$$

We define $J_\mu \equiv (8\pi G)\nabla^\nu T_{\nu\mu}$ and assume unimodular gravity integrability $dJ = 0$.

Then we use Bianchi identities and obtain

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{R}g_{\mu\nu} + (\Lambda_0 + \int_I J)g_{\mu\nu} = 8\pi G\mathbf{T}_{\mu\nu}$$

where Λ_0 is a constant of integration, $\Lambda = \Lambda_0 + \int_I J$, I is a one dimensional path from some reference event. We could say that the energy-violation current J is the source of a term satisfying the dark energy equation of state. The final result after computation of the current J_ν is that

$$\Lambda \approx 4\alpha\Lambda_{obs},$$

where Λ_{obs} is the observed value of the cosmological constant.

Group field theory and phantom energy

The energy density ρ_ψ satisfies the conservation equation

$$\dot{\rho}_\psi + 3H(1+w)\rho_\psi = 0,$$

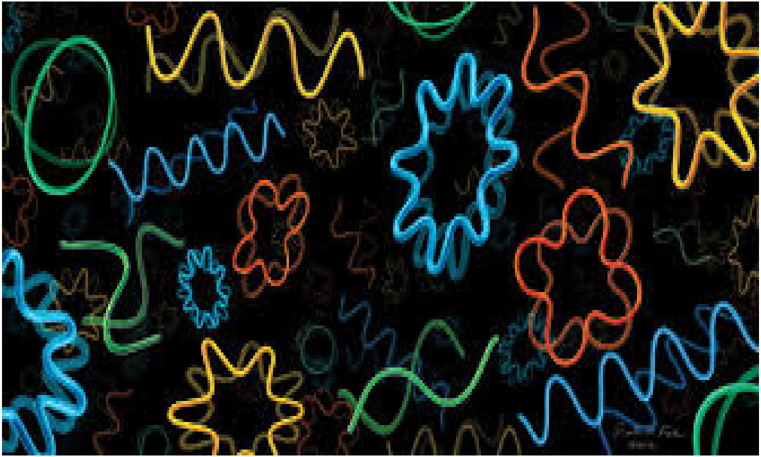
where $H = \frac{\dot{V}}{3V}$ and we take $V = a^3$ for scale factor a. Then it is possible to rewrite the equation as

$$\frac{d\rho_\psi}{dV} + \frac{1+w}{V}\rho_\psi = 0.$$

We get a solution for constant w

$$\rho_\psi = \frac{\rho_{\psi_0}}{V^{1+w}}.$$

The key result of GFT is that phantom energy could model dark energy, (X. Pang, D. Oriti, 2105.03751).



Metastring theory

Authors of arXiv: 1905.09463, P. Berglund, T. Hübsch and D. Minic, showed that one can successfully address the problem of dark energy and the observed de Sitter space-time in a generic, non-commutative generalized geometric phase space formulation of ST. What we have in our minds is a model with the underlying chiral world-sheet Hamilton's action for the strings:

$$S_{string} = \frac{1}{4\pi}\int_{\Sigma}(\partial_\tau \mathbb{X}(\eta_{AB} + w_{AB})\mathbb{P}^B - \mathbb{P}^A H_{AB}\mathbb{P}^B)\, d^2\sigma,$$

where $\mathbb{X}^A(\tau,\sigma) = (X^a(\tau,\sigma), \widetilde{X}_a(\tau,\sigma))$ combine the sum (x^a) and the difference ($\tilde{x}_a$) of the left- and right-movers on the string and $\mathbb{P}^A = \partial_\sigma \mathbb{X}^A$ are closely related to the generalized momenta. The mutually compatible dynamical fields w_{AB}, η_{AB} and H_{AB} are the anti-symmetric symplectic structure, the symmetric polarization metric and the doubled symmetric metric.

This new framework for ST based on quantum space-time shows the essential quantum non-locality of any quantum theory. The result of the computations is that the three scales associated with the cosmological constant M_Λ, the Planck units M_P and the effective particle physics scale M are related by a formula

$$M_\Lambda \sim \frac{M^2}{M_P}$$

and the conclusion is that this reformulation of string theory leads naturally to a positive cosmological constant.

String gas cosmology

One of the possibilities how to model the accelerated expansion in the last 5 billion years in string theory is by **string gas cosmology** (SGC). It is interesting that we could explain in SGC **the existence of three large dimensions**. Other important thing is that **the cosmological singularities could be abolished** . Because SGC are fully non-singular, one should expect that **some condition of the classical cosmic singularity theorem is violated** . This is a consequence of **uncoventional spacetime geometry** , because spatial sections have the topology of a three-torus.

The **null energy condition** is generally fulfilled, if for every future pointing null vector field k^μ holds

$$T_{\mu\nu}k^\mu k^\nu \geq 0$$

at every point of the spacetime, where $T_{\mu\nu}$ is the energy-momentum tensor. It means that any observer of a cosmological spacetime with total energy density ρ and pressure p should find that $\rho + p \geq 0$.

The **avoidance of singularities in SGC** entails certain costs: the **null energy conditions** are violated in the earliest stages; It is an open question in string theory whether it allows the null energy condition violations, even of the "effective" type, when the full non-perturbative theory is taken into account. However, it is clear that the era of null energy condition violation must be very brief, it must come to an end and it should be replaced by the inflationary era (Brett McInnes, arXiv:hep-th/0502209).

String cosmology based on AdS/CFT correspondence

When we model the accelerated expansion **the strong energy condition** (SEC) is then necessary violated. The main surviving condition is **the dominant energy condition** (DEC), which gives us

$$\rho > |p|.$$

We know today that the data is consistent with $w = -1$, but they are certainly consistent with lower values. If this is confirmed, it will lead to the violations of DEC.

Strong energy condition is fulfilled, if for every timelike vector field X^a holds

$$(T_{ab} - \frac{1}{2}Tg_{ab})X^aX^b \geq 0$$

in every point of the spacetime.

We mentioned that there are numerous **objections to the phantom cosmologies.** First of all $w < -1$ leads to **the violation of causality.** If we want to see how it would be possible, let's consider local speed of sound in a perfect fluid, which is given by $\sqrt{\frac{dp}{d\rho}}c$. Fluids in which pressure dominates density are in danger having **speed of sound which will exceed that of light** (Brett McInnes, JHEP 0208:029). But the cosmological equation of state is applicable only to strictly isotropic situations and therefore p and ρ do not depend on spatial position, only on time. Hence it could not serve as a full equation of state and we could not deduce anything from it alone about the speed of sound.

Completely different objection is that the violation of DEC frequently leads to **cosmological singularities of a peculiarly unpleasant kind**. We assume now according to (R.R. Caldwell, astro-ph/9908168) that the phantom energy comes to dominate after a matter-dominated period ends at $t = t_m$. The formula for the scale factor is

$$a(t) = a(t_m)[-w + (1 + w)\frac{t}{t_m}]^{\frac{2}{3(1+w)}}.$$

w is constant and spatial sections are supposed to be flat. We see immediately that the scale factor is singular at the finite proper time

$$t = (\frac{w}{1 + w})t_m.$$

It follows that the spacetime is not asymptotically de Sitter but it comes to an end in finite time and it is destroyed by an excessive expansion. We have a formula for the density

$$\rho = K[-w + (1+w)(\frac{t}{t_m})]^{-2}.$$

As the universe evolves towards this strange disaster, its density has to increase. The final state is singular, with "infinite" density and pressure . We call it **Big Smash cosmology.**

Construction of phantom cosmologies without Big Smash

We start with the formulation of the continuity equation

$$\frac{d\rho}{da} = -3\frac{\rho + p}{a},$$

from which follows that in every phantom cosmology ρ must increase with $a(t)$. The only possibility how we would avoid the Big Smash is that ρ approaches a constant, let's say $\frac{3}{8\pi L^2}$. Then

$$\rho = \frac{3}{8\pi L^2} - S(a),$$

where $S(a)$ is a positive function which decreases monotonically to zero as $a(t)$ tends to infinity.

The pressure composes from two parts, which correspond to the de Sitter universe and some matter ingredient:

$$p = p_{dS} + p_{\psi}$$

We could rewrite it as following:

$$p = -\rho_{dS} + w_{\psi}\rho_{\psi} = w_{\psi}\rho - \frac{3}{8\pi L^2}(1 + w_{\psi})$$

ρ given by one of the previous equations corresponds to total density and it gives us an appropriate form of the equation of state in asymptotically de Sitter phantom cosmology. It is important that **it is only for non-constant** w **that a Big Smash can be averted.** The superposition of the two fluids, when each of them have constant but different w, does not have itself the constant w.

Now, since we have $\rho_\psi + p_\psi = \rho_\psi(1 + w_\psi) < 0$ and ρ_ψ is negative and p_ψ is positive, we must have $-1 < w_\psi < 0$. Hence, there is **no reason to expect that its sound speed will exceed the speed of light.** Since Λ dark energy has strictly constant density and pressure, and therefore cannot influence the speed of signal propagation, the combined fluid does not violate causality.

We will denote $\gamma = 3(1 + w_\psi)$, where $0 < \gamma < 3$. We could rewrite the equation of state as

$$p = -(1 - \frac{\gamma}{3})\rho - \frac{\gamma}{8\pi L^2}$$

and $\rho + p = -\frac{\gamma}{3}S(a)$. It is clear that γ **measures the extent to which the DEC is violated** and we suppose that γ is a small parameter, so that the violation is not too robust.

We obtain after some elementary computations the solution for the scale factor

$$a(t) = \cosh^{(\frac{2}{\gamma})}(\frac{\gamma t}{2L}).$$

Then the metric is

$$g(\gamma, A) = -dt \otimes dt + A^2 \cosh^{(\frac{4}{\gamma})}(\frac{\gamma t}{2L})(d\theta_i \otimes d\theta_i).$$

These metrics are the simplest non-singular phantom metrics, which tend to de Sitter space in its flat slicing.

An approach based on non-commutative geometry

It is possible to formulate a different approach to QG based on the ideas of **non-commutative geometry** (Tejinder P. Singh, arXiv: 1911.01955). Universe is made of **atoms of space-time-matter** (STM) according to this approach. Dark energy consists of about 10^{122} STM atoms and each have an associated **mass** of $10^{-33} eV/c^2$. We call this particle **mitron** and it is a quantum gravitational entity.
Mitrons have a **very long wave length**, and their kinetic energy T is negligible compared to their potential energy V. We could essentially compute the **pressure** as $T - V$ and the **energy density** as $T + V$. This implies that we obtain an equation of state as for the cosmological constant, $p = -\rho$ and therefore acceleration of the expanding universe.

Asymptotic safety program

Swiss cheese model, which combines globally the homogeneous and isotropic Universe and locally a Schwarzschild black hole:

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2(\theta) d\phi^2)],$$

where κ has values -1,0 or 1. We define further a radius r_Σ, where r_Σ is a constant. The spacetime will be described for $r \leq r_\Sigma$ by a static spherically symmetric metric given in Schwarzschild-like coordinates by

$$ds^2 = -J(R)F(R)dT^2 + \frac{dR^2}{F(R)} + R^2[d\theta^2 + \sin^2(\theta)d\varphi^2],$$

where J and F is bigger than 0 (F. K. Anagnostopoulos, S. Basilakos, G. Kofinas, V. Zarikas, arXiv:1806.10580).

Entropic gravity

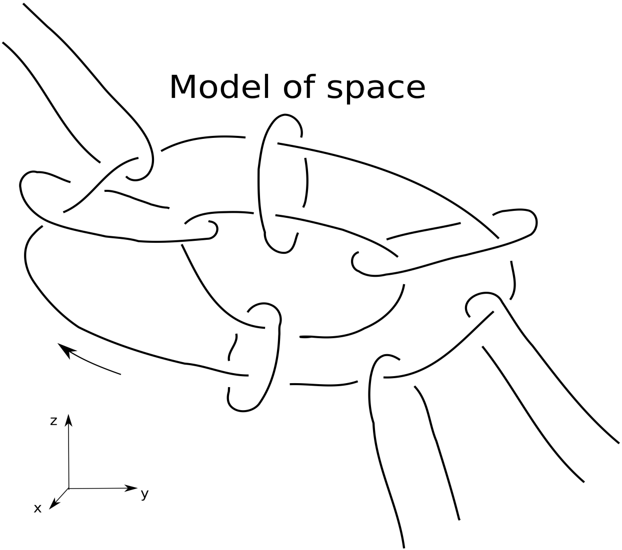
- our universe contains a large amount of quantum information in extremely long range correlations of the underlying microscopic degrees of freedom
- the present local laws of physics are not capable of detecting or describing these delocalized states.
- De Sitter space behaves as a glassy system with a very high information density that is slowly being manipulated by the microscopic dynamics

Details are in E. Verlinde, arXiv:1611.02269.

Necessity of discretization

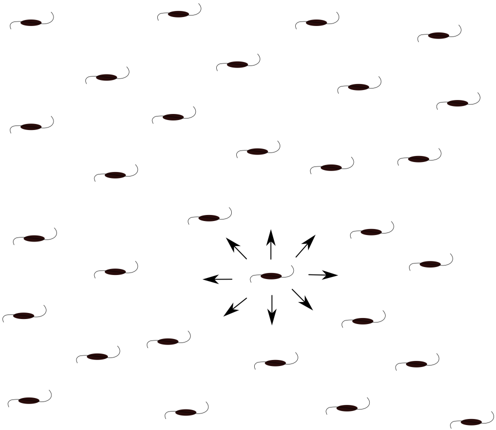
’In any case, it seems to me that the alternative continuum-discontinuum is a genuine alternative; i.e. there is no compromise here. In theory there cannot be space and time, only numbers. It will be especially difficult to elicit something like a spatio-temporal quasi-order from such a schema. I can not picture to myself how the axiomatic framework of such a physics could look. But I hold it as altogether possible that developments will lead there.’ [Albert Einstein]

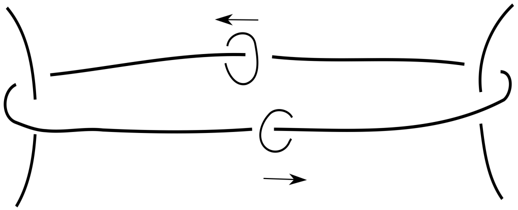
Ring paradigm



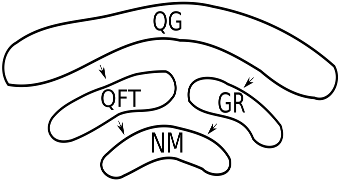
Postulate

All particles and fields in Nature are attached to some ring. They could move only on these rings, which are non-local objects.



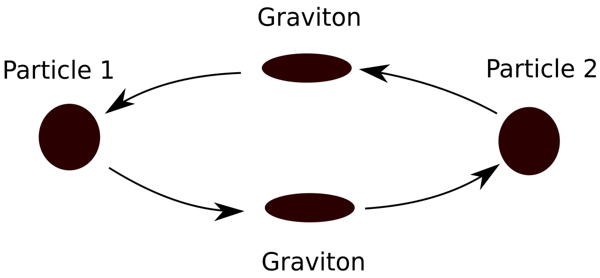


Limits

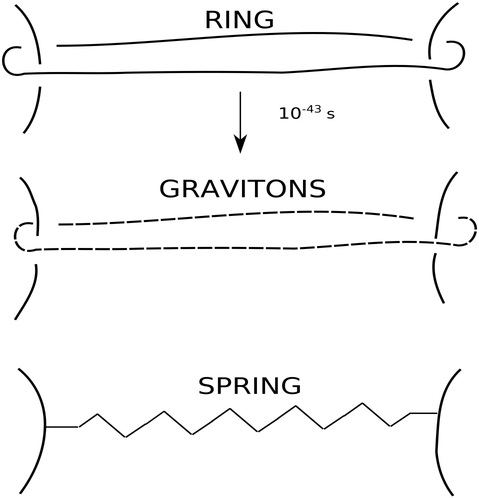


1. we quantize longitudinally vibrating rings and we obtain graviton as a phonon; particle sector is not changed
2. the second limit is general relativity

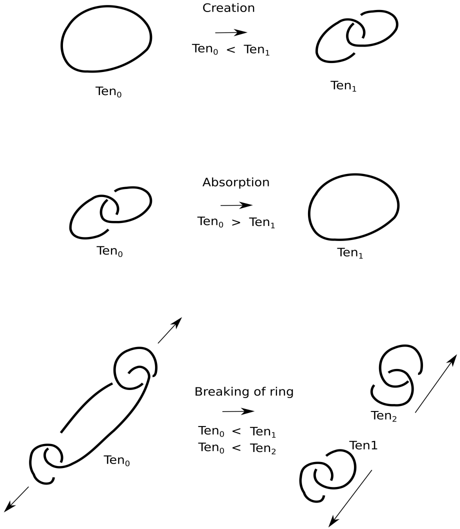
Old concept of interaction in gravity sector

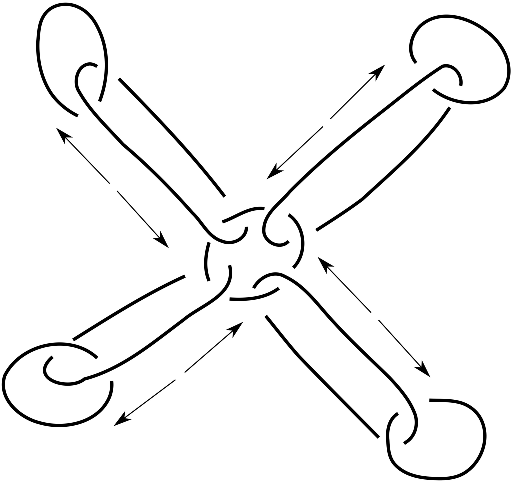


Graviton as a phonon

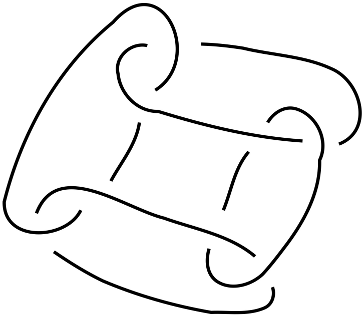


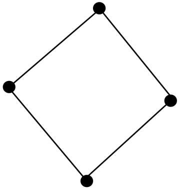
Model of dark energy in ring paradigm





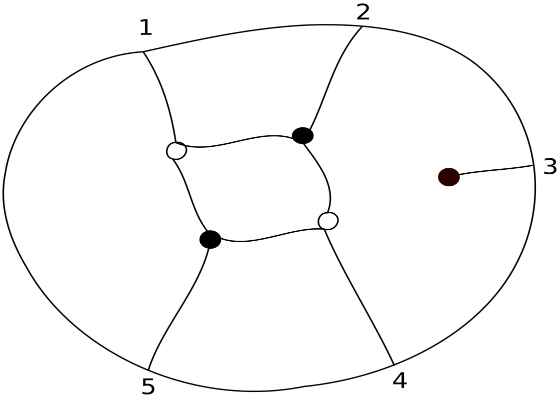
Mathematical problem: we have a finite collection of N rings S^1 in R^3, which could not touch; Give a **complete characterization of all non-homeomorphic structures**, which could be constructed from this finite collection of rings. Every two rings could be linked only once, they could not be knotted or twisted (in the case we have differentiable structure). We do not consider any Brunnian type of link in 3 and more rings.





Definition

A **plabic graph** is an undirected planar graph G, which we draw inside a disk (considered modulo homotopy) with n boundary vertices on the boundary of the disk, labeled $1, \ldots, n$ in clockwise order, as well as some colored internal vertices. These internal vertices are strictly inside the disk and are each colored either black or white. Each boundary vertex i in G is incident to a single edge. If a boundary vertex is adjacent to a leaf (vertex of degree 1), we refer to that leaf as a lollipop.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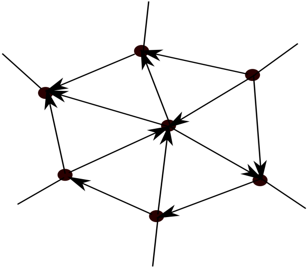
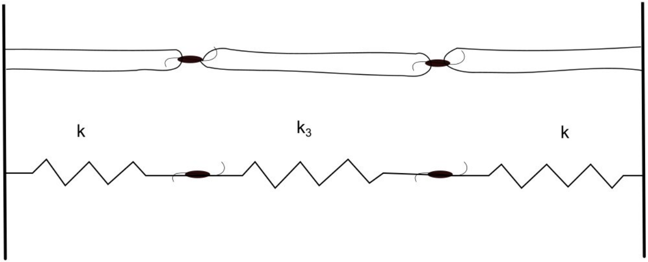


Toy model



Hamiltonian is equal to

$$\hat{H} = \sum_{i=1}^{2} \frac{1}{2m} \hat{P}_i^2 + \sum_{i=1}^{2} V_{ij} \hat{Q}_i \hat{Q}_j,$$

where

$$V_{ij} = \begin{pmatrix} \frac{1}{2}k + \frac{1}{2}k_3 & -\frac{1}{2}k \\ -\frac{1}{2}k & \frac{1}{2}k + \frac{1}{2}k_3 \end{pmatrix},$$

and we will compute eigenvalues of the Hamiltonian:

$$w_1 = \sqrt{\frac{k}{m}}, w_2 = \sqrt{\frac{k}{m} + 2\frac{k_3}{m}}.$$

The key observation is that the supermassive black holes are vibrating with temperature T_G.
The next step is to construct the partition function Z and the average energy of the system

$$E = \sum_{\alpha} E_\alpha P_\alpha,$$

where $P_\alpha = \frac{\exp(-\beta E_\alpha)}{Z}$ and $\beta = \frac{1}{k_B T_G}$.
The result is

$$E = \hbar \frac{e^{-2\beta\hbar(w_1+w_2)}}{(1-e^{-\beta\hbar w_1})^2(1-e^{-\beta\hbar w_2})^2}\left[\frac{w_1}{(1-e^{-\beta\hbar w_1}} + \frac{w_2}{1-e^{-\beta\hbar w_2}}\right]$$

Field quantization

$$L(\psi, \dot{\psi}) = \frac{1}{2} \int \dot{\psi}(x)\dot{\psi}(x)\, d^3x - \frac{1}{2} \int \int K(x - x')\psi(x)\psi(x')\, d^3x' d^3x,$$

where $K(x - x') = K(x' - x)$.

$$H = \frac{1}{(2\pi)^3} \int \hbar w(k) a^+(k) a(k)\, d^3k$$

$$P = \frac{1}{(2\pi)^3} \int \hbar k a^+(k) a(k)$$

We are searching for the $\frac{dP}{dt}$ and we want to put it equal to the Newtonian force.

List of solved problems

- cosmological singularities
- singularities in black holes
- EPR paradox
- Feynman path integral

Open questions

1. We know that general relativity is perturbatively non-renormalizable. Will be the results of computation for interaction of graviton (phonon) with other particles finite?
2. How will change the Ring paradigm early Universe cosmology? The singularities in black holes disappeared, because we define the theory on a grid, which is ever-present.
3. How will change the Ring paradigm the view on determinism of physical theories? Because a trajectory (gravitational ring) is prepared first and then the particle is traveling on it. It also means that the RP works with two velocities: velocity of light c and some other velocity for creation of these rings $c_g > c$; The lower bound on this velocity is at least 10^{70} m/s. But we couldn't send any information by it.

Conclusion

- it is possible that the problem of dark energy is solvable only in quantum gravity
- we suggested a new approach (ring paradigm), which has advantages in cosmology
- ring paradigm is connected with very nice mathematics (algebraic geometry and topology)

’It doesn’t matter how beautiful your theory is, it doesn’t matter how smart you are. If it doesn’t agree with an experiment, it’s wrong.’ [Richard P. Feynman]

Some pictures were taken from web, some were created by software Inkscape.

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